Math 472: Computer Assignment 3 — due Monday, Nov.7, 2005

1. Consider the IVP
$$y'(t) = -y(t) + 2e^{-t}\cos(2t), \quad y(0) = 0$$
with exact solution $y(t) = e^{-t}\sin(2t)$.
(a) Write a Matlab program based on Euler's method that uses Richardson extrapolation to obtain a method with higher accuracy.
Test your program on the interval $[0, 10]$ with a stepsize $h = 0.1$ (and $h/2 = 0.05$). Compare your answers to the solution obtained with the basic Euler method using stepsizes of h and $h/2$. Display the errors for all three methods in one common plot.
(b) Write a Matlab program based on Euler's method that uses Richardson extrapolation to obtain a method with adaptive stepsize.
Test your program on the interval $[0, 10]$ with an initial stepsize $h = 0.1$. Use a value of $\delta = 10^{-2}$ for the stepsize control. Compare your answer to the solution obtained with Euler's method using a *fixed* stepsize of $h = 0.1$.
Plot both the (exact) errors and the error estimate based on Richardson extrapolation discussed in class.

2. Consider a restricted form of the three-body problem in which a body of small mass orbits two other bodies with much larger masses, such as a satellite or a space ship orbiting the earth-moon system. We will use a two-dimensional Cartesian coordinate system in the plane determined by the three bodies. We fix the x-axis through the two centers of mass of the larger bodies. The origin is fixed at the center of mass of the larger bodies. So the coordinate system is rotating! Thus, we need to take into account gravitational as well as centrifugal and Coriolis forces. The resulting equations of motion are obtained by solving the following system of ODEs for the satellite-earth-moon system
$$\begin{aligned} x''(t) &= x(t) + 2y'(t) - \mu_p \frac{x(t)+\mu_m}{d_m} - \mu_m \frac{x(t)-\mu_p}{d_p} \\ y''(t) &= y(t) - 2x'(t) - \mu_p \frac{y(t)}{d_m} - \mu_m \frac{y(t)}{d_p} \end{aligned}$$
with
$$\mu_m = 0.0121486,\ \mu_p = 1-\mu_m,\ d_m = \Big((x(t)+\mu_m)^2 + y^2(t)\Big)^{3/2},\ d_p = \Big((x(t)-\mu_p)^2 + y^2(t)\Big)^{3/2}.$$
The coordinates (x, y) are the coordinates of the satellite. The value μ_m is the proportion of the mass of the moon to the mass of the entire system. The unit of length is the mean distance between earth and moon (about 384000 km), the unit of time is a month. The values d_m and d_p denote the cubes of the distances of the satellite from the moon and earth, respectively. The initial values
$$x(0) = 0.994,\ x'(0) = 0,\ y(0) = 0,\ y'(0) = -2.1245$$
are chosen to produce a(n almost) periodic orbit.
(a) Find the period of the orbit.
(b) Plot the orbit.
Here periodicity means that the satellite has the same position and speed at time $t = T$ as at the initial time $t = 0$.
Hint: Use Matlab event handling for the built-in ODE solvers to determine the period. In particular, show that the stopping criterion
$$stopvalue(t, y) = \boldsymbol{d}(t)^T \boldsymbol{v}(t)$$

with

$$\boldsymbol{d}(t) = [x(t) - x(0), y(t) - y(0)]^T,$$

the vector from the current point to the initial point, and $\boldsymbol{v}(t) = \boldsymbol{d}'(t)$ the current velocity is an appropriate stopping criterion.