

Math 478 — Homework Assignment 2, due Feb. 8, 2007

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1. Use the Peano kernel theorem to obtain the following well-known formula for *Simpson's rule*:

$$\int_0^2 f(x)dx = \frac{1}{3} [f(0) + 4f(1) + f(2)] - \frac{1}{90} f^{(4)}(\xi).$$

2. (a) Write the following system of initial value problems

$$\begin{aligned}y'' + yz &= 0, & y(0) &= 1, & y'(0) &= 0 \\z' + 2yz &= 4, & z(0) &= 3\end{aligned}$$

as a system of first-order initial value problems.

- (b) Convert the following system of higher-order time-dependent ODEs into a system of first-order equations that do not explicitly depend on  $t$ :

$$\begin{aligned}x''' - 5tx''y'' + \ln(x')z &= 0 \\y'' - \sin(ty) + 7tx'' &= 0 \\z' + 16ty' - e^t zx' &= 0.\end{aligned}$$

Hint: introduce an additional differential equation for  $t$ .

3. Use the same method applied to prove Theorem 1.21 (as well as Theorem 1.2 in the Iserles book) to prove convergence of the *implicit midpoint rule*

$$\mathbf{y}_{n+1} = \mathbf{y}_n + h\mathbf{f}\left(t_n + \frac{1}{2}h, \frac{1}{2}(\mathbf{y}_n + \mathbf{y}_{n+1})\right).$$

4. Find the order of the implicit midpoint rule.