Math 478 — Homework Assignment 6, due April 26, 2007

1. Provide the details of the case $j = 0$ in the formula

$$D_{j0} = \frac{d}{dt}\left[\operatorname{sinc}\frac{t\pi}{h}\right]_{t=t_j=jh} = \begin{cases} 0, & j = 0 \\ \frac{(-1)^j}{jh}, & \text{otherwise,} \end{cases}$$

   for the entries in the $k = 0$ column of the spectral differentiation matrix D on unbounded grids.

2. Derive the formula

$$D^{(2)}_{j0} = \frac{d^2}{dt^2}\left[\operatorname{sinc}\frac{t\pi}{h}\right]_{t=t_j=jh} = \begin{cases} -\frac{\pi^2}{3h^2}, & j = 0 \\ 2\frac{(-1)^{j+1}}{j^2h^2}, & \text{otherwise,} \end{cases}$$

   for the entries in the $k = 0$ column of the second-order differentiation matrix $D^{(2)}$ on unbounded grids.

3. In the periodic case, determine the Fourier differentiation matrices D_N, D_N^2, and $D_N^{(2)}$ for $N = 2$ and $N = 4$, and confirm that in both cases $D_N^2 \neq D_N^{(2)}$. Note: D_N^2 is the square of D_N, while $D_N^{(2)}$ is the differentiation matrix that corresponds to the second-order derivative as in Problem 2 above.

4. Find the linear transformation that is needed to map the Chebyshev points from the standard interval $[-1, 1]$ to an arbitrary interval $[a, b]$.