RANDOMNESS: what is that and how to cope with it (with view towards financial markets)

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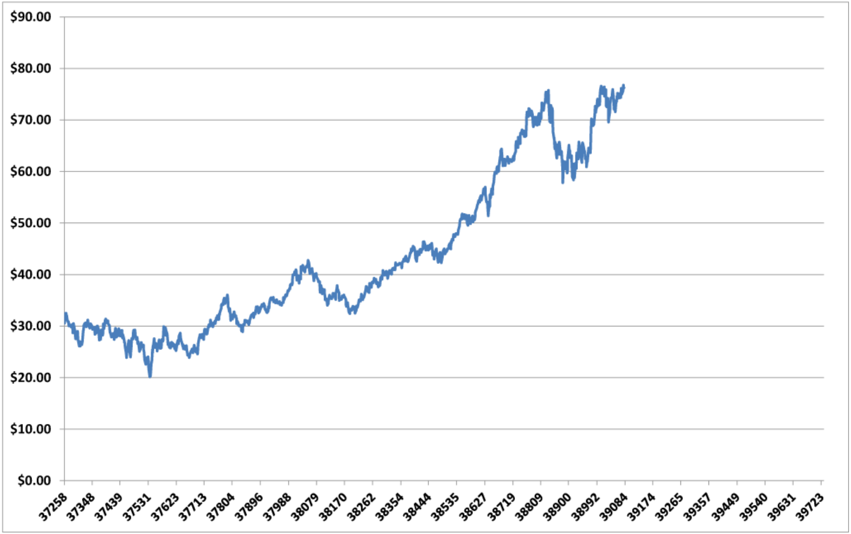
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Randomness is almost everywhere

Modeling it (the randomness) is FUN

What's randomness

- **Event(s) with Random Outcomes**

  Random, Stochastic, Uncertain, Chaotic, Unpredictable

- **Examples of Random Events:**

  flip a coin, temperature next Friday at noon, Dow Jones Industrial Average Tomorrow at 3:40pm, moving of a car in traffic, etc

- **Deterministic Outcomes:** - flipped coin, temp yesterday, number of days in a year 2089, etc

- "Almost Random" - small noise in deterministic system

"Probability, science originated in consideration of games of choice, should become the most important object of human knowledge"
Pierre Simon, Marquis de Laplace, 23 April 1749 - 5 March 1827, France

What is random and what is not

- More a philosophical question

  causality, predetermined/unknown future, all odds are known/uknown

- Difficult to distinguish Luck from Skills, Forecast from Prophecy

  weather in Chicago, spam of predicting the market, the most talented CEOs/actors (survivorship bias)

- Easy to "predict" the past but "almost impossible" to predict the future

- Rolling a die (gambling in casino) and stock price are very different type of randomness

  gambling - the rules are known, the sources of randomness are known

  stock market - the risk and randomness are changing, the rules and factors are unknown, we can only assume something about the randomness (the distribution of uncertainty)

- An attempt to describe various types of randomness "The Black Swan" by N.N.Taleb;

- David Aldous book review

  http://www.stat.berkeley.edu/~aldous/157/Books/taleb.html

- Andrew Gelman book review

  http://andrewgelman.com/2007/04/nassim_talebs_t/

What parts of mathematics study randomness?

- Probability
- Statistics

... and what's the difference?

Both study the same objects and phenomena, but from very different points of view.

... an example will help to see the difference

Simplest example

Flip a coin

- The outcomes Head (H) or Tail (T)
- Chances of H and T

(a) say equal, 50/50, fair coin
or (b) $\mathbb{P}(H) = p, \ \mathbb{P}(T) = 1 - p$, for some fixed and known $p \in (0, 1)$

This is a probabilistic model of flipping a coin.

Probability Theory assumes the coin (the distribution) is known, and tries to find/predict/study something about future observed events.
It is a “transparent” or “open” box.

Problem: you play a game in which you are paid \$5 if H and \$3 if T. How much should you pay to enter the game?
Answer: In a fair game you should pay the expected wining sum
$\mathbb{E}(\text{payoff}) = 5 \cdot p + 3 \cdot (1 - p)$

Flip a coin ... con't

- The model is done
- You can find about anything related to this model

  Flip the coin many times, look at the number of heads, number of consecutive heads, first time you have N heads and M tails, etc. All these probabilities can be evaluated

- Some of the quantities of interest can be found by probabilistic methods (using in particular combinatorics) or by simulations
- You do not need a coin to simulate the game (computer can do)
- Computer Simulated Outcomes for flipping a coin

  $p = 0.7$ H H T H H H H T H H H H T T H H H

  $p = 0.1$ T T T H T T T T T T T T T T T T T T T T T T T T H T T T T T T T T T H T T T T T T T T T T T T T T H T T T T T T T H T T T T H T T

  $p = 0.5$ - fair coin H H T T T H H T H H T T H H T T T T H T T H T T H

- More on flipping a coin by Prof. Persi Diaconis

  http://news.stanford.edu/news/2004/june9/diaconis-69.html

Same coin but a different question

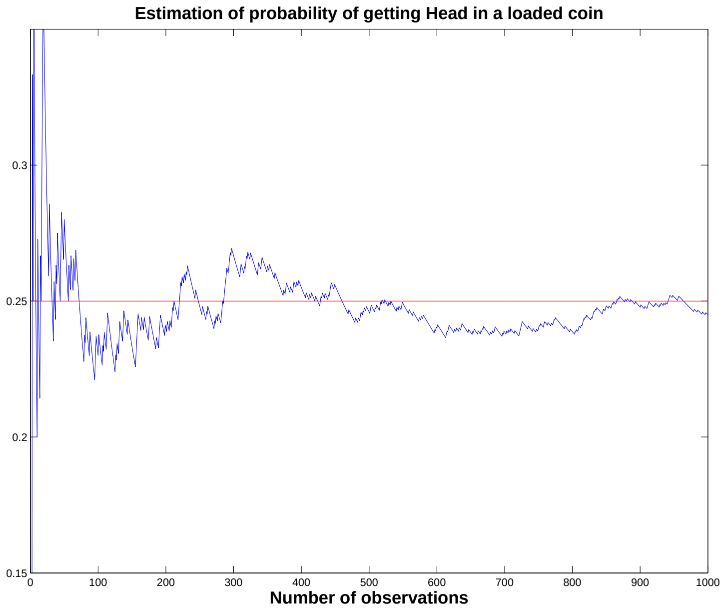
Problem: You are given a coin. Find if this coin is fair or loaded.

- the mathematical question: What is $p = \mathbb{P}(H)$?
- Answer: We can not find it exactly, but we can estimate it. How?
- Well, what's p? Chances that H will appear, or probability that H will appear. Hence

$$\widehat{p} = \frac{\#\text{ of Heads}}{\#\text{ of total observations}}$$

- More observation, better estimates (law of large numbers)

Statistics - based on past observations we try to find/inffer/estimate the probabilities of some events to happen. We try to make sense of past data.


Estimation of probability of getting Head in a loaded coin
0.3
0.25
0.2
0.15
0
100
200
300
400
500
600
700
800
900
1000
Number of observations

Same "game" ... same type of models ... same questions, same methods

- Roll a die and get paid the face value
  the model: six faces, six outcome $\Omega = \{1, 2, 3, 4, 5, 6\}$. Each face ends up with some probability $p_1, p_2, \ldots, p_6$. Note $p_1 + p_2 + \ldots + p_6 = 1$.
- Fair value to enter the game? Expected payoff
  $\mathbb{E}(\text{payoff}) = 1 \cdot p_1 + 2 \cdot p_2 + \ldots + 6 \cdot p_6$

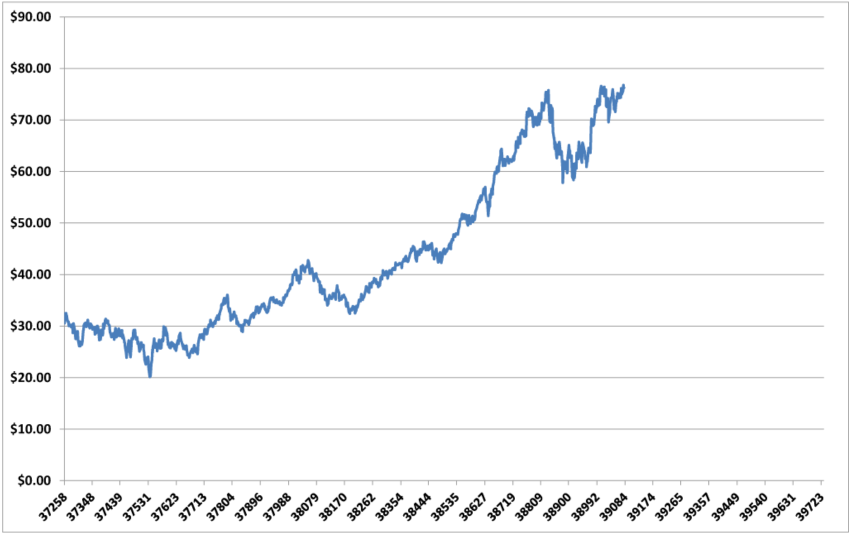
  Fair die, then $p_1 = p_2 = \ldots = p_6 = 1/6$ and $\mathbb{E}(\text{payoff}) = 3.5$
  Simulations 2 3 5 5 2 2 3 3 1 2 6 2 1 3 4 5 6 2 2 4 5 6 2 3 1

Other Casino type games. Same idea, as long as the rules are known.

- Roulette? Easy, a fair die with 36 faces
- Blackjack? Also "easy", just more complicated combinatorics. No independency, so one can count the cards

Back to financial markets

predicting the stock price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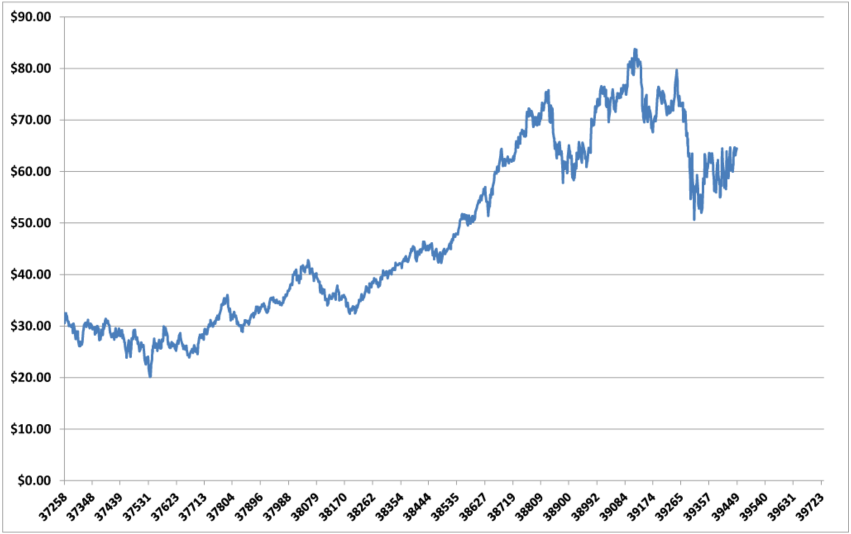
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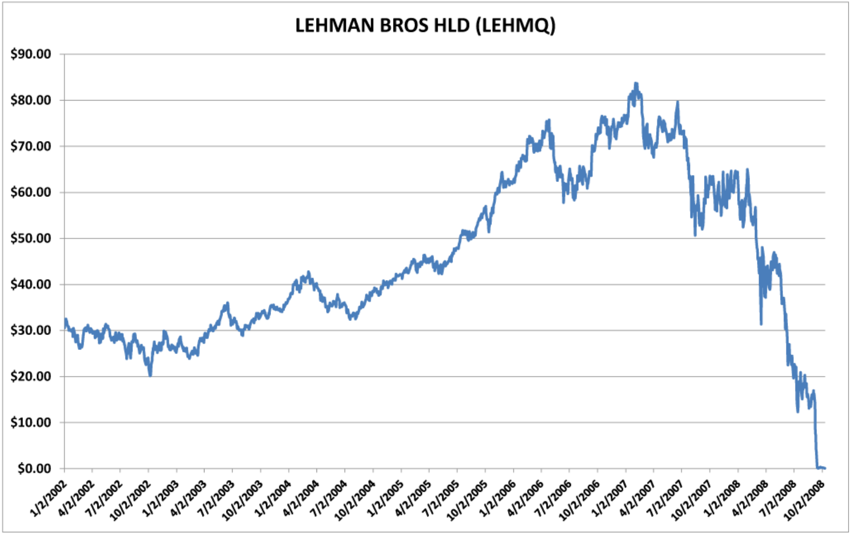
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What is so different in financial markets?

- The rules, sources of randomness, and sources of risk are changing.
- The factors driving the randomness in the market are unknown; we can only assume some properties about them (e.g. distribution).
- The stock price today already reflects all the past information. The price is based on **demand and supply**.
- Nobody can predict (with certainty) the future stock price.

HOWEVER!

still many things can be done

Fundamental Law

No Arbitrage or No Free Lunch
(can not make money for sure out of nothing)

Example (of arbitrage):
Bank ABC: deposit at 3.5% and borrow at 3.8% per year
Bank XYZ: deposit at 3% and borrow at 3.4% per year

Arbitrage: borrow, say $10,000 from XYZ, and deposit into ABC. This costs $0 at initiation. Close out the position at the end of the year, and get a sure profit of $10.

Disclaimer: of course, we assumed that ABC and XYZ will not default within one year

Hedging/Replication of derivative contract

- Bank PQR wants to buy today the following (future) contract: for no \$'s down today, to agree on a price of $\$K$, paid in one year, for getting one share of AAPL (Apple Inc) also in one year.

- Bank KLM wants to sell this contract. Assume that KLM has access to credit (can borrow) for 3.0% per year.

- **Question:** What is $\$K$ that KLM wants to charge PQR?

- **Answer:** The fair price $K = \$563.4718$.

Hedging/Replication of derivative contract

$$K = \text{`AAPL price today'} \times (1 + 0.03) = \$547.06 \times 1.03 = \$563.4718$$

- Why? Because KLM can replicate. Assume that KLM enters the contract.
  - Borrow $\$547.06$ for one year under 3%
  - Buy one share of AAPL
  - Zero cost today
  - In one year ....
  - Get $K = 563.4718$ from PQR in exchange for that share of AAPL
  - Return to the lender exactly $\$563.4718$ (which is initial borrowing of $\$547.06$ plus the interest of $\$16.4118$)

Simple complex case - modeling stock price

Idea: Stock price - a banking account, but random (why not?)
Banking account $B_t = B_0 \cdot e^{rt}$, with r - interest rate

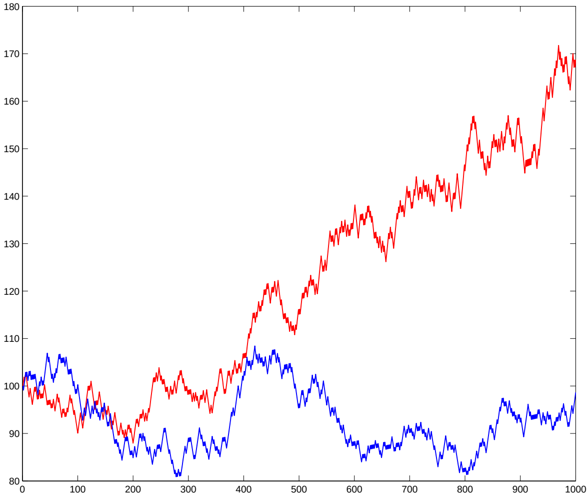
$$B_{t+\Delta t} = B_t e^{r\Delta t}$$

Stock - a random banking account, kind of ...

$$S_{t+\Delta t} = S_t e^{\mu\Delta t \pm \sigma\sqrt{\Delta t}}$$

with equal probabilities up or down ($\pm$).

Parameters μ, σ implied from the market or estimated historically.



Simulation of stock price using Black-Scholes-Merton model .

Modeling randomness in real life

What if the rules are unknown? What if 'the die' is changed, and 'the casino' does NOT tell us that?

Examples: financial markets, temperature anomalies, turbulence etc

How to model?

- Make simplifications
- Start from simple
- Keep track of general rules and laws of 'nature'
- Use past data, but do not overuse it
- If no explicit solution, simulation usually helps

Thank You !

The end of the talk ...
but not of the story