# An Overview
# of
# Radial Basis Functions

by

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## REFERENCES

There are more references in [Hardy '90] and in [Golberg & Chen '96]. Those in [Hardy '90] not mentioned below are mainly concerned with special applications of multiquadrics, those in [Golberg & Chen '96] not covered below are from the application of radial basis functions in the dual reciprocity method for boundary element methods, and from the area of *kriging* in geostatistics.

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