Why Mathematical Modeling for Undergrads?
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A separate modeling course for undergraduates would serve two important purposes -

(a) students would learn the modeling process, the basis of all modern applied math, and understand applied math as the thought-process and tool-box for the study of real-world problems. Most of our courses emphasize only the tool-box (i.e., the methods) aspect of itwhile this course would emphasize the modeling and design process. The lectures, HWs, and particularly the project help train the students in the study of real-world problems.

(b) students (including those from other departments) would have better appreciation for the wide scope of Applied Mathematics, and the exciting advanced courses that we offer here at IIT that would be beneficial to them (both intellectually and for their future careers). Some of the students could be inspired to participate in the MCM/ICM contest which is held every Spring semester.

Experiences based on teaching Math 486 in Spring 2014:

Many students were very excited to learn so many new topics, and asked me for recommendations for courses at IIT that covered those topics in more detail. An oft-repeated comment was "I wish I had taken this course before I took .....", the point being that they would have been more motivated and would have understood the importance of the underlying mathematics, how it applies by itself and in conjunction with other math that they have learned. I feel that this course works as a 300-level version of Math 100, in the sense that it gives a thorough overview of all parts of Applied Mathematics and presents them as one coherent perspective to understanding a variety of real-life phenomenon.

As many students pointed out to me, many 100 and 200-level math courses tend to be boring and unmotivated (from students' perspective) which leads to very few students continuing with math courses or becoming math majors. A version of this course can help overcome that perception of mathematics and better prepare our students not just for upper-level math courses, but also for applying mathematical thinking in the real-world.

The 8-10 week project would be a major part of learning in this course. What I wanted from the projects in Math 486 was to force students to think holistically about sparsely described real-world problems and then utilize their comprehensive mathematical knowledge to formulate, modify, justify, and analyze models (including the needed data) for the study of their chosen problem. You can read more about my expectations and advice in the attached document.

The projects topics were adapted from the MCM and ICM contests from the past 15 years - see http://www.comap.com/undergraduate/contests/mcm/ The students had to form groups of 2-3 students each and choose topic from among 14 topics I wrote for them.

I was very pleased that all the projects were completed to my satisfaction. Of course, not all of them were exceptional, but all of them were meaningful and it was obvious that a lot of thought and effort went into each of them.

Many undergrad students mentioned that unlike other departments, math courses tend to lack interesting and challenging projects, and they were very pleased by the effort they had to put into this project. They compared it to a capstone/ honors project that many other Math departments require their majors to take.

MATH 486/ MATH 522: Course Project Instructions

The project is an important part of this course - not just in terms of the grade, but for the sake of comprehensive, practical understanding of how to apply modeling framework to an open ended real-life problem. This is why the problem statements that I have given to you are just short and open-ended descriptions of the certain real-life situations. You have complete freedom in mathematical interpretation of the problem and how you "solve" it. The only requirement is that you use the mathematical modeling process, and justify your model and its conclusions as they apply to the problem. Its a test of your creativity in formulation of models and solution methods, and your ability to find and understand relevant mathematical knowledge.

Here are some important suggestions to keep in mind as you choose and work on a project problem.

1. **Break the problem into three main parts**: What are you modeling and how can you formulate it in mathematical terms? What ideas for algorithms do you have? How will you compare algorithms/ models? Think about how you will use specific mathematical tools (our textbook has a pretty long list such mathematical tools - just look at the table of contents) to describe and solve each of these aspects. Discuss with your team members. Note that there could be more than one way for approaching each of these parts.

2. Based on your discussion in step 1, you should have some **preexisting mathematical techniques/ methods/ models** that you want to focus on. First look up your textbooks (from this course and from your other math courses), and then go to the library (in person or online) to look up textbooks and papers that discuss the mathematics of your approach, and to learn as much about your (interpretation of the) problem as possible. What approaches have others considered for similar problems?

Note that you are allowed to use older research as your motivation (with proper attribution and reference, of course). But you can not simply plagiarize (exactly or after some minor modification) other's work/ ideas. Such plagiarism is fairly easy to catch through online search and will result in zero credit for your project in addition to any further disciplinary action by the University.

3. Re-discuss Step 1 after your literature survey for relevant mathematical methods/ models. What are reasonable assumptions and what are not? What is a reasonable Mathematical model for problems of this kind and what is not? **Finalize your mathematical model** and keep track of all the simplifying assumptions you are making for it. Don't worry if this is a simple model as you can modify this model as needed later on.

4. **How will you "solve" this model?** Can you use a standard algorithm or do you have to write your own algorithm combining other algorithms? You might have to write your own computer code that incorporates appropriate solvers from Mathematica/ Matlab. In any case, you have to use a computer program to solve your model.

5. Do you need **input data** to solve this model? Where can you get this data? Search online for data depositories. If nothing is available, then discuss how you can create your own "synthetic data" (data that does not come from real-life but is created, possibly with help of a computer program, to mimic real-life data). You have three possibilities here: (1) Use preexisting data that fits your needs (with proper reference as always); (2) Modify preexisting data to fit your

needs (explain how and why); (3) Write a mini-model incorporating the properties of the data you need, and then use this model/rules generate your own data on a computer.

6. **Get solutions** from your model and data. Note the plural - you must aim for multiple solutions so that you can compare them and pick the best. How well does your solution answer the project question? Now, you should go through the whole modeling process again, and possibly "unsimplify" your model to get better results. Analyze the strengths and weaknesses of your modeling approach, and **modify your model** to eliminate as many of these weaknesses as possible.

7. Quantitatively and qualitatively compare the various solutions and come up with your **conclusions**. Think about what are the best ways to display and discuss your solution data and conclusions.

8. Put together the **project report** and all that you need for it - description of the models, data, algorithms/ programs, tables and graphs of the output solutions, references to textbooks and papers that you have used, etc.

Your typed project report should be 15 to 25 pages long (in 11pt or 12pt font), excluding the appendix. It should include the following sections:

1. Introduction (This should give a 1-2 page summary of all that follows int he report, including your final conclusions. You can have it as introduction in the beginning of the report or as summary at the end of the report.)

2. Statement and analysis of the Problem (Quote the problem statement and then restate it in your own words, describing how and what you interpret the problem to be and how you are going to approach solving it. Also describe the results of your literature survey about how others have modeled and solved same or similar problems (with references of course) and how your interpretation is similar to and different from them.)

3. Description of the Model (List and discuss the assumptions of your model. Discuss how you designed it and your justification for the model. List all the variables/ parameters used in the model. Note that you could have more than one model here if your approach is to compare results from more than one model)

4. Analysis and Testing of the Model (List/Describe/present your input data and output results; and the conclusions from your model. Discuss how you tested the model, including, if relevant, the error analysis and stability of your solutions (if you change some of the data/ parameters, does that change your solution drastically?). You should give an step-by-step outline and detailed description of any algorithms/ computer programs that you are using, explaining their purpose and intent. Standard mathematical algorithms can be referenced through textbooks but you should still give a short description of what they do.)

6. Results and Quality of the Model (Discuss your results and analysis, with graphs, figures, etc. to make your final conclusions. And also discuss, how you would improve the models you studied - in particular the strengths and weaknesses of your model(s).)

7. References (You have to explicitly list all sources of your Mathematical tools, models, algo-

rithms, data, etc. This should be as extensive as possible.)

8. Appendix (include computer programs that you wrote, numerical calculations, proofs, anything relevant you did not include in the main report.)

In addition to your textbooks from this course and all other Math/CS courses that you have taken, here are some sources for important algorithms for fundamental Mathematical models/ tools:

1. Numerical Recipes: The Art of Scientific Computing; William H. Press and others.

2. Numerical Methods that Work; Forman Acton.

3. Introduction to Algorithms; Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein.

4. Operations Research: Applications and Algorithms; Wayne Winston.

You may not need them for actual programming since you can utilize appropriate solvers from Matlab/ Mathematica, but they can serve as sources of ideas for your computations.