The Worst Way To Compute π

Hemanshu Kaul, kaul@iit.edu
Department of Applied Math, Illinois Tech

On the occasion of π Day, I wanted to share with you something surprising I learned about π within the last year, and hope that some of you find this new and unexpected too. Here goes.

There are many methods for computing the digits of π. Classically, π was defined geometrically (e.g. as area of a circle of unit radius, or as the circumference of circle of unit diameter) and geometrical ideas were used to find an approximate value of π using regular polygons to inscribe and circumscribe a circle. In modern times, Taylor series formula for arctan along with Machin-type formulas* have been most commonly used for efficiently finding more and more digits of π, reaching up to 1.2 trillion digits. Other faster converging power series for π based on Ramanujan's mystifying formulas for π † have pushed the records to 50 trillion digits of π in January 2020!

But what about bad methods for computing π? Ones that take a lot of time and effort to generate just a few digits of π.

The famous Buffon's needle experiment says: If a needle of length D is dropped N times on a surface on which parallel lines are drawn $2D$ units apart, and it comes to rest crossing any line X of those times, then we may approximate π by N/X (this is based on the fact that the probability of crossing a line in one instance of this experiment is $1/\pi$; look it up). This sets up a Monte-Carlo algorithm for approximating π which is too slow to use in practice. It has been estimated that we would need to drop a needle 10000 times before we could reliably estimate the first two digits of π. But remember we like that. We are ambitious, so we ask "Can we do worse?" Something worse than dropping a needle countless times!

In 2003, G. Galperin described a method for finding digits of π that might be unbeatable under our perverse criterion. Imagine two solid cubes, let's call them A and B, resting on a frictionless surface with an immovable wall to their left. We push B to the left towards A. Obviously a series of collisions are going to ensue. Assume no energy is lost during these collisions. If A and B each weigh 1 kg, then how many collisions (including those between A and B, and those with the wall) will there be in total? First, B strikes A, transfers all its energy to A, which now moves towards the wall, strikes it, and moves back towards B, finally striking B which moves off towards right, never to return. This gives a total of 3 collisions. What if A is 1 kg and B is 100 kgs, how many collisions will

*see http://en.wikipedia.org/wiki/Machin-like_formula
†see http://en.wikipedia.org/wiki/Chudnovsky_algorithm

we have in that scenario (always under these ideal conditions)? It turns out we will have exactly 31 collisions. Hmmmm.... That's interesting! Would 1kg vs 10000 kgs give us 314 collisions!?!?

Galperin showed that if A weighs 1 kg and B weighs 10^{2m} kgs then the number of collisions will be the integer equal to the first $m+1$ digits of π.[‡]

So, to calculate first 30 digits of π (a very modest aim), we need to record the collisions between a body of mass 1 kg and a body of mass 10^{60} kgs. That's greater than the mass of the observable universe by several orders of magnitude! Computing just 14 digits would require the equivalent of counting collisions between a 1 kg ball and Jupiter! Even if we change the units from kg to grams, we are still in trouble! To say nothing of the time it would take to record the collisions!

Epilogue: This discussion has a fascinating epilogue. Last year, Adam Brown, a physicist, described an isomorphism between Galperin's experiment and Glover's famous quantum search algorithm! An unexpected connection between π, Dynamics, and quantum computing! Read this article to learn more about quantum search and this connection: `http://www.quantamagazine.org/how-pi-connects-colliding-blocks-to-a-quantum-search-algorithm-20200121/`
.

A π day bonus: Look up the book '*Not A Wake: A Dream Embodying π's Digits Fully For 10000 Decimals*' by Mike Keith. It is claimed to be the first book ever written completely in πlish, that peculiar dialect of English in which the numbers of letters in successive words follow the digits of the number π. Divided into ten sections of 1000 digits, each written in a different style (from poems to screenplays) its words "spell out" the first 10,000 digits of π'. Just fascinating!

[‡]Galperin (see `http://www.maths.tcd.ie/~lebed/Galperin.%20Playing%20pool%20with%20pi.pdf`) proved this under the assumption that following conjecture about digits of π is true: for every N, the first $2N$ consecutive digits of π do not contain a string of $N-1$ consecutive 9s in the N digits in the right half. He proves this conjecture for N up to 10^8, and with very high probability for all large N.