Math 478 — Homework Assignment 3, due Feb. 22, 2007

1. Derive explicitly the three-step Adams-Moulton and Adams-Bashforth methods.

2. Which of the following multistep methods is convergent?

    (a) $\boldsymbol{y}_{n+2} - \boldsymbol{y}_n = h\left[\boldsymbol{f}(t_{n+2}, \boldsymbol{y}_{n+2}) - 3\boldsymbol{f}(t_{n+1}, \boldsymbol{y}_{n+1}) + 4\boldsymbol{f}(t_n, \boldsymbol{y}_n)\right]$,

    (b) $\boldsymbol{y}_{n+2} - 2\boldsymbol{y}_{n+1} + \boldsymbol{y}_n = h\left[\boldsymbol{f}(t_{n+2}, \boldsymbol{y}_{n+2}) - \boldsymbol{f}(t_{n+1}, \boldsymbol{y}_{n+1})\right]$,

    (c) $\boldsymbol{y}_{n+2} - \boldsymbol{y}_{n+1} - \boldsymbol{y}_n = h\left[\boldsymbol{f}(t_{n+2}, \boldsymbol{y}_{n+2}) - \boldsymbol{f}(t_{n+1}, \boldsymbol{y}_{n+1})\right]$,

    (d) $\boldsymbol{y}_{n+2} - 3\boldsymbol{y}_{n+1} + 2\boldsymbol{y}_n = h\left[\boldsymbol{f}(t_{n+2}, \boldsymbol{y}_{n+2}) + \boldsymbol{f}(t_{n+1}, \boldsymbol{y}_{n+1})\right]$,

    (e) $\boldsymbol{y}_{n+2} - \boldsymbol{y}_n = h\left[\boldsymbol{f}(t_{n+2}, \boldsymbol{y}_{n+2}) - 3\boldsymbol{f}(t_{n+1}, \boldsymbol{y}_{n+1}) + 2\boldsymbol{f}(t_n, \boldsymbol{y}_n)\right]$,

3. Determine the order of the three-step method

$$\boldsymbol{y}_{n+3} - \boldsymbol{y}_n = h\left[\frac{3}{8}\boldsymbol{f}(t_{n+3}, \boldsymbol{y}_{n+3}) + \frac{9}{8}\boldsymbol{f}(t_{n+2}, \boldsymbol{y}_{n+2}) + \frac{9}{8}\boldsymbol{f}(t_{n+1}, \boldsymbol{y}_{n+1}) + \frac{3}{8}\boldsymbol{f}(t_n, \boldsymbol{y}_n)\right],$$

the *three-eighths* scheme. Is it convergent?

4. The formula

$$\boldsymbol{y}_{n+2} = (1-\alpha)\boldsymbol{y}_{n+1} + \alpha\boldsymbol{y}_n + \frac{h}{12}\left[(5 - \alpha)\boldsymbol{f}(t_{n+2}, \boldsymbol{y}_{n+2}) + 8(1 + \alpha)\boldsymbol{f}(t_{n+1}, \boldsymbol{y}_{n+1}) + (5\alpha - 1)\boldsymbol{f}(t_n, \boldsymbol{y}_n)\right]$$

is known to be exact for all polynomials of degree m or less for all α. Determine α so that it will be exact for all polynomials of degree $m + 1$. Find α and m.