Math 478 — Homework Assignment 5, due April 3, 2007

1. Find the solution at $t = \frac{1}{2}$ of the linear two-point boundary value problem

$$y''(t) + 2y'(t) + 10t = 0$$
$$y(0) = 1, \quad y(1) = 2$$

   by applying the finite difference method (by hand) with $h = \frac{1}{2}$.

2. Consider the linear boundary value problem

$$y''(t) = u(t) + v(t)y(t) + w(t)y'(t)$$
$$a_0 y(a) + a_1 y'(a) = \alpha, \quad b_0 y(b) + b_1 y'(b) = \beta.$$

   Set up the resulting system of linear equations if the finite difference method is used with meshsize $h = \frac{b-a}{m+1}$. Make sure that you use only $\mathcal{O}(h^2)$ approximations.

3. Consider the eigenvalue BVP $y''(t) = \lambda y(t)$ with $y(-1) = y(1) = 0$.

   (a) Show that the eigenvalues and eigenfunctions of this problem are given by

$$\lambda = -\left(\frac{n\pi}{2}\right)^2, \qquad \sin\frac{n\pi}{2}(x+1), \qquad n = 1, 2, \ldots.$$

   (b) Describe an algorithm you would use to solve this problem with the finite difference method.