Math 578 — Homework Assignment 2, due Feb. 8, 2007

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1. Use the Peano kernel theorem to obtain the following well-known formula for *Simpson's rule*:

$$\int_0^2 f(x)dx = \frac{1}{3}\left[f(0) + 4f(1) + f(2)\right] - \frac{1}{90} f^{(4)}(\xi).$$

2. (a) Write the following system of initial value problems

$$\begin{aligned} y'' + yz &= 0, \quad y(0) = 1, \quad y'(0) = 0 \\ z' + 2yz &= 4, \quad z(0) = 3 \end{aligned}$$

   as a system of first-order initial value problems.

   (b) Convert the following system of higher-order time-dependent ODEs into a system of first-order equations that do not explicitly depend on t:

$$\begin{aligned} x''' - 5tx''y'' + \ln(x')z &= 0 \\ y'' - \sin(ty) + 7tx'' &= 0 \\ z' + 16ty' - e^t zx' &= 0. \end{aligned}$$

   Hint: introduce an additional differential equation for t.

3. Present a detailed discussion of the end of the proof of convergence of Euler's method (equations (25) and (26)).

4. Use the same method applied to prove Theorem 1.21 (as well as Theorem 1.2 in the Iserles book) to prove convergence of the theta method

$$\boldsymbol{y}_{n+1} = \boldsymbol{y}_n + h\left[\theta \boldsymbol{f}(t_n, \boldsymbol{y}_n) + (1-\theta)\boldsymbol{f}(t_{n+1}, \boldsymbol{y}_{n+1})\right] \qquad \theta \in [0,1].$$

5. Given $\theta \in [0,1]$, find the order of the method

$$\boldsymbol{y}_{n+1} = \boldsymbol{y}_n + h\boldsymbol{f}\left(t_n + (1-\theta)h, \theta\boldsymbol{y}_n + (1-\theta)\boldsymbol{y}_{n+1}\right).$$