Math 578 — Homework Assignment 4, due March 8, 2007

1. By considering only the autonomous scalar equation $y' = f(y)$, prove that the explicit Runge-Kutta method with the tableaux

$$\begin{array}{c|cccc} 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ \hline & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} \end{array}$$

   is of order four.

2. Write the theta method

$$\boldsymbol{y}_{n+1} = \boldsymbol{y}_n + h\left[\theta \boldsymbol{f}(t_n, \boldsymbol{y}_n) + (1-\theta)\boldsymbol{f}(t_{n+1}, \boldsymbol{y}_{n+1})\right]$$

   as a Runge-Kutta method.

3. Determine all values of θ such that the theta method is A-stable.

4. Is there any reason to distrust the following numerical scheme for solving the IVP $\boldsymbol{y}' = f(t, \boldsymbol{y})$

$$\boldsymbol{y}_{n+3} - 3\boldsymbol{y}_{n+2} + 2\boldsymbol{y}_{n+1} = h\left[\boldsymbol{f}_{n+3} + 2\boldsymbol{f}_{n+2} + \boldsymbol{f}_{n+1} - 2\boldsymbol{f}_n\right]?$$

   Explain.