Math 578 — Homework Assignment 5, due April 3, 2007

1. Verify that the function $y(t) = c \sin t$ is a solution of the boundary value problem

$$\begin{gathered} y''(t) + y(t) = 0 \\ y(0) = 0, \quad y(\pi) = 0 \end{gathered}$$

for any constant c. Comment.

2. Find the solution at $t = \frac{1}{2}$ of the linear two-point boundary value problem

$$\begin{gathered} y''(t) + 2y'(t) + 10t = 0 \\ y(0) = 1, \quad y(1) = 2 \end{gathered}$$

by applying the finite difference method (by hand) with $h = \frac{1}{2}$.

3. Consider the linear boundary value problem

$$\begin{gathered} y''(t) = u(t) + v(t)y(t) + w(t)y'(t) \\ a_0 y(a) + a_1 y'(a) = \alpha, \quad b_0 y(b) + b_1 y'(b) = \beta. \end{gathered}$$

Set up the resulting system of linear equations if the finite difference method is used with meshsize $h = \frac{b-a}{m+1}$. Make sure that you use only $\mathcal{O}(h^2)$ approximations.

4. Consider the eigenvalue BVP $y''(t) = \lambda y(t)$ with $y(-1) = y(1) = 0$.

   (a) Show that the eigenvalues and eigenfunctions of this problem are given by

$$\lambda = -\left(\frac{n\pi}{2}\right)^2, \qquad \sin \frac{n\pi}{2}(x+1), \qquad n = 1, 2, \ldots.$$

   (b) Describe an algorithm you would use to solve this problem with the finite difference method.