Supplier Commitment & Production Decisions Under a Forecast-Commitment Forecast

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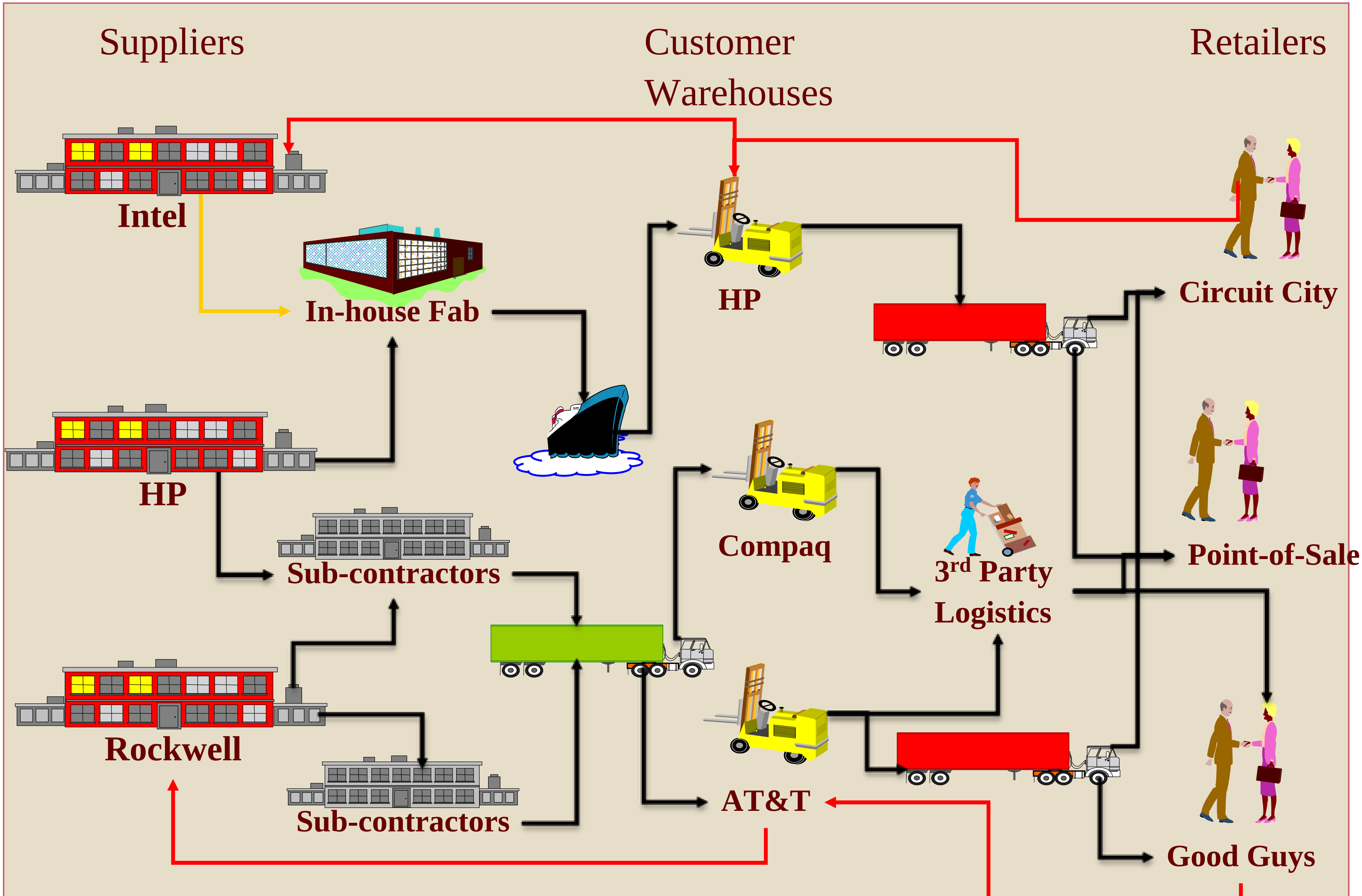
Overview

- Background & Problem Motivation
- Taxonomy of Supply Contracts Literature
- Operational Model of Production Planning with Commitments under a Forecast-Commitment Contract
  - Single Product, Single Customer Problem
    - Model Formulation
    - Results & Managerial Insights
    - Effect of a Capacity Constraint
- Related Research
  - Customer Problem of Forecast Selection
- Concluding Remarks

Background: Supply Chain Management

- What is a Supply Chain?
  - A network of facilities that perform the functions of:
    - Procurement of Materials
    - Transformation of Materials
    - Distribution of Finished Goods to Customers

- Supply Chain Management
  - Logistics of Controlling Material and Information flows



Example of a Supply Chain in High-technology Industry

Problem Motivation

- Worked with Application Specific Integrated Circuits (ASICs) Manufacturer
  - ICs embedded in other divisions' products
- Originally: Internal supplier and cost center
  - Capacity allocation strategy set at corporate level
- At the time: Profit center with internal customers
  - Allocation based on maximizing division's profit (myopic)
  - Transitioning: Wholly-owned Subsidiary; mix of internal and external customers (customer relationship management!)
- Conflicting Short-term and Long-term Goals

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Problem Motivation (cont.)

- ASICs market is extremely volatile — To gain a competitive advantage
  - Manufacturer willing to make commitments against 1-period-ahead forecasts (viewed as strategic weapon)
  - Alternative to point-system for prioritizing customers under development
- Contracts are negotiated as new product generations are introduced. Contracts specify:
  - Prices, production, holding and penalty costs
- Research Goal: Develop commitment and capacity allocation models that incorporate existing business practices

Contract Structure

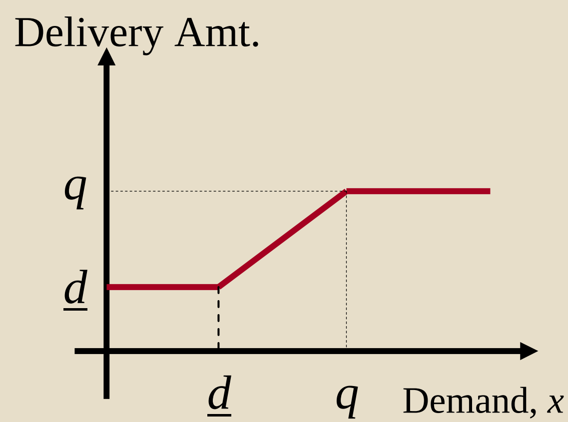
- One period in advance:
  - Customer provides order forecast, $\bar{f}$
  - Supplier makes a commitment to the customer based on forecast, C
  - Supplier decides on a production quantity, q

- When orders are placed:
  - Customer bound to order a fraction of forecast, α, unless supplier committed to a lesser value, $\underline{d} = \min(\alpha \bar{f}, C)$

    Let $x \equiv$ demand

    Customer takes delivery of

    $$= \min(q, \max(x, \underline{d}))$$



Contract Structure: Penalty Scheme

- **Commitment Penalty**: for failing to commit up to the forecast

$$\pi_1 \cdot (\text{Forecasted Amount - Commitment})^+$$

- **Delivery Penalty:** for not delivering on commitment amount

$$\pi_2 \cdot (\text{min (Demand, Committed Amount) - Units Supplied})^+$$

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Supplier's Problem

Goal: choose commitment and production quantity that maximize profits

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Taxonomy – Brief Overview

- Different Contract Types:
  - Buy-Back Contracts (Pasternack 1985)
    - Full price returns for partial order, or partial refund for all returns
  - Pay-to-Delay (Brown and Lee 1999)
    - Fixed-fee upfront
  - Contracts with Options (Barnes-Schuster et al. (2002))
    - Apparel industry – Two period Model
  - Revenue-Sharing Contracts (Lariviere and Cachon (2005))
    - Video industry – Supply Chain Coordination
  - Push, Pull and Advance Purchase Contracts (Cachon 2004)
    - Push (Price-only), Pull (Vendor-Managed Inventory) – Allocation of Inventory Risk
  - Quantity Flexibility (Tsay 1999)
    - Customer agrees to purchase a fraction of forecast
    - Supplier agrees to supply up to a fraction above

Supplier’s Problem

Goal: choose commitment and production quantity that maximize profits

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Notation

w Wholesale price
c Variable production
α Fraction of forecast that customer is obligated to purchase
$\underline{\underline{d}}$ Minimum required order quantity
f Customer forecast provided to supplier
x R.V. for the customer demand in the current period
$f_S(x \mid f)$ Conditional density of customer demand given forecast (f)

Decision Variables:

q Production quantity

C Amount commited to the customer

$\Pi_S(q, C, f)$ *is the supplier's profit function given customer's forecast*

Formulation – Supplier's Problem

Supplier's Profit Maximization Problem:

$$\Pi_s(q,C,f)=\max_{C,q}\begin{cases} w\left[\overbrace{\int_{x=0}^{\underline{d}}\underline{d}\,f_s(x)dx+\int_{x=\underline{d}}^{q}xf_s(x)dx+\int_{d=q}^{\infty}qf_s(x)dx}^{\text{Revenue}}\right] \\ -cq \quad \longleftarrow \text{Production Cost} \\ -\pi_1[f-C]^+ \quad \longleftarrow \text{Commitment Penalty} \\ \underbrace{-\pi_2\left[\int_{x=q}^{C}(x-q)f_s(x)dx+\int_{x=C}^{\infty}(C-q)f_s(x)dx\right]}_{\text{Customer Satisfaction/Delivery Penalty}} \end{cases}$$

Solution Approach

- Want C and q that maximizes $\Pi_S(q, C, f)$

1. For all production quantity values, q, we find the optimal commitment value
   - $C^*(q) = \arg\max_C [\Pi_S(q,C, f)]$
2. Given $C^*(q)$, we find q that maximizes $\Pi_S(q, C^*(q), f)$.

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Optimal Commitment Response — Results

- **Theorem:** *An optimal supplier commitment response,* $C^*(q)$, *for a given production quantity, q, and the customer forecast, f, is*

$$C^*(q) = q \text{ or } \bar{f}.$$

That is, the supplier will either commit to the amount to be produced or commit to the amount forecasted by the customer.

- **Lemma:** *The supplier's optimal commitment,* $C^*(q)$, *is non-decreasing in the amount to be produced, q.*

Find Optimal Production Quantity, given Commitment Response, $\max_q \Pi_S(q, C(q), f)$.

- When $C(q) = q$. For $\alpha f \leq q < f$ the value function is

$$\Pi\left(q, C(q)=q, f\right) = -cq + w\left[\int_{x=0}^{\alpha f}(\alpha f) f_S(x)dx + \int_{x=\alpha f}^{q} x f_S(x)dx + \int_{x=q}^{\infty} q f_S(x)dx\right] - \pi_1[f-q]$$

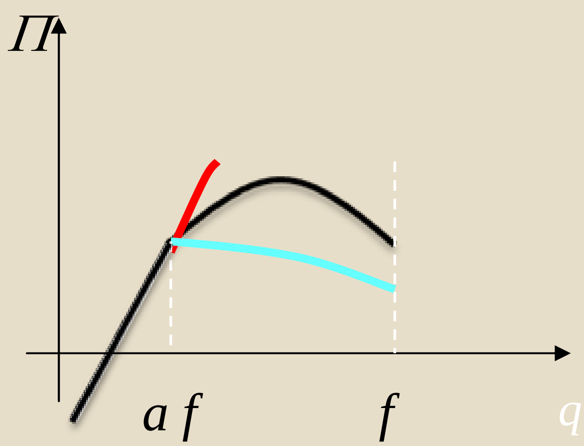
- The first and second derivatives are:

$$\frac{\partial \Pi(q,q,f)}{\partial q} = -c + w[1 - F_S(q)] + \pi_1$$

$$\frac{\partial^2 \Pi(q,q,f)}{\partial q^2} = -w f_S(q) \leq 0$$

- with stationary point

$$\hat{q} = q_C^* = F_S\left(\frac{w + \pi_1 - c}{w}\right)$$



- So $\Pi_S(q,q,f)$ can be concave, concave increasing or decreasing in q.

Finding q^* under $C(q) = q$ (cont.)

- For $f \le q$ the value function is

$$\Pi(q,q,f) = -cq + w\left[\int_{x=0}^{\alpha f}(\alpha f) f_S(x)dx + \int_{x=\alpha f}^{q} x f_S(x)dx + \int_{x=q}^{\infty} q f_S(x)dx\right]$$

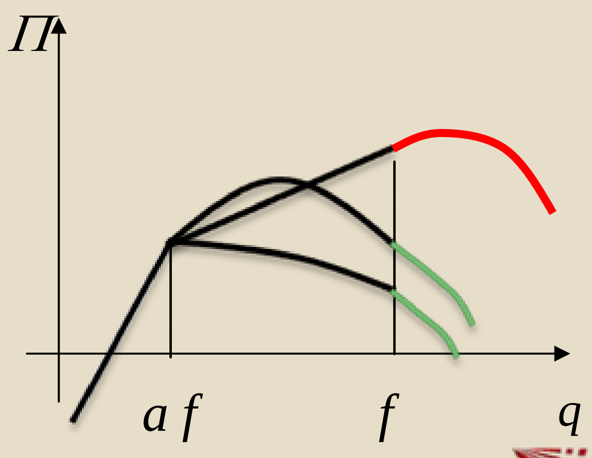
- The first and second derivatives are:

$$\frac{\partial \Pi(q,q,f)}{\partial q} = -c + w[1 - F_S(q)]$$

$$\frac{\partial^2 \Pi(q,q,f)}{\partial q^2} = -w f_S(q) \le 0$$

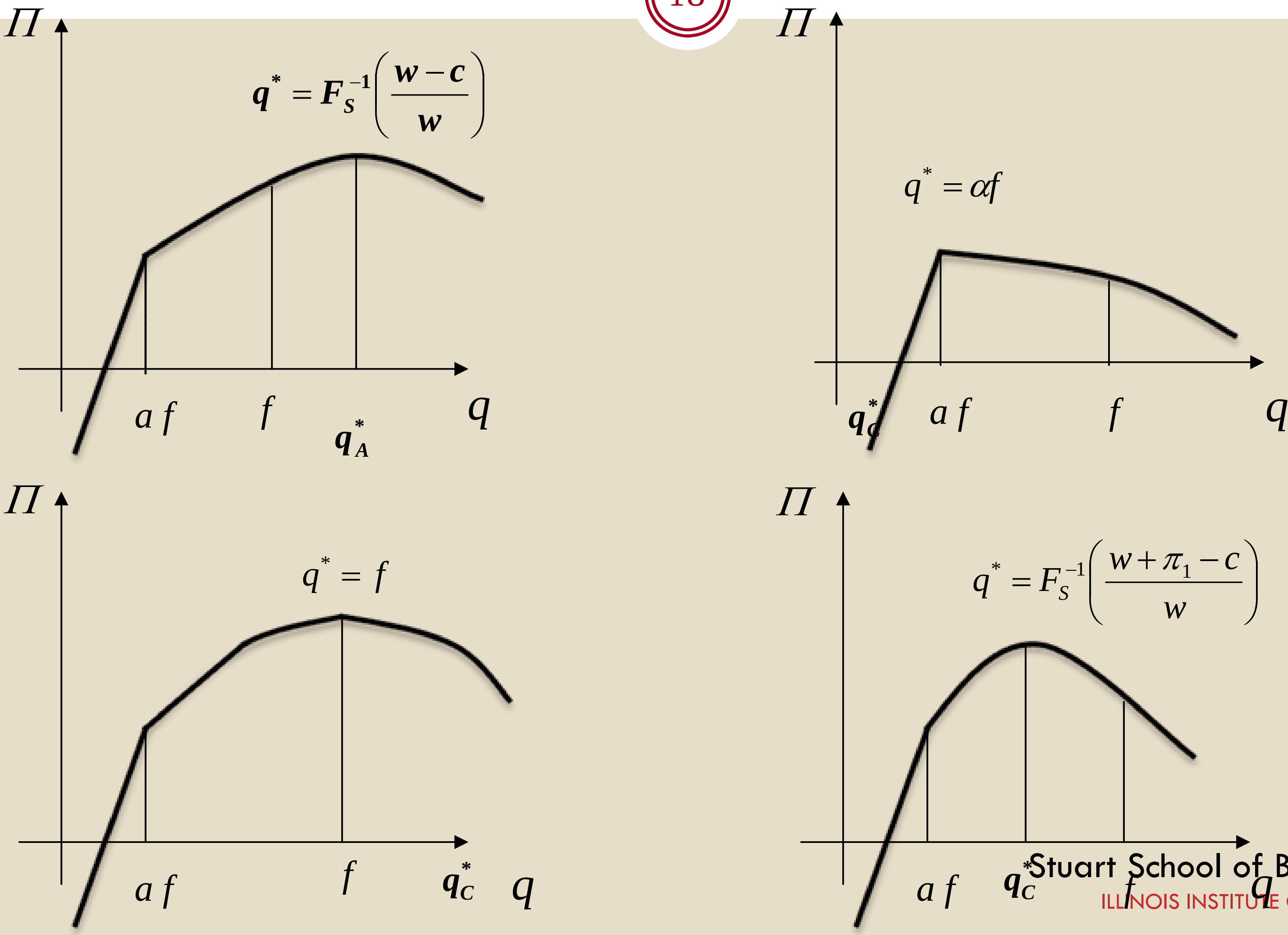
- with stationary point

$$\hat{q} = q_A^* = F_S^{-1}\left(\frac{w-c}{w}\right)$$



- So $\Pi_S(q, q, f)$ can be concave, or concave decreasing in q.

Forms of Value Function, $C(q) = q$



Finding q^* under strategy $C(q) = f$

- If the supplier's commitment strategy is to commit to the forecasted amount, the optimal production quantities are:

$$q^* = \begin{cases} \alpha f \text{ or } f \text{ or } F_S^{-1}\left(\dfrac{w+\pi_2-c}{w+\pi_2}\right) & \alpha f \le q < f \\ f \text{ or } F_S^{-1}\left(\dfrac{w-c}{w}\right) & f \le q \end{cases}$$

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Related Problems

- **Problem A:** Supplier not liable for any shortages

$$q_A^* = F_s^{-1}\left(\frac{w-c}{w}\right)$$

- **Problem B:** Supplier is fully liable for any shortages

$$q_B^* = F_s^{-1}\left(\frac{w+\pi_2-c}{w+\pi_2}\right)$$

- **Problem C:** Supplier liable only for commitment amount

$$q_C^* = F_s^{-1}\left(\frac{w+\pi_1-c}{w}\right)$$

Optimal Policy Depends on these Critical Values

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Optimal Commitment and Production Quantity Policy

- **Theorem:** *The optimal commitment and production quantity pair for the supplier is:*

$$\left(C^*, q^*\right) = \underset{\substack{\left(f, q_A^*\right), \left(f, q_B^*\right), \left(q_C^*, q_C^*\right) \\ (f, f), (\alpha f, \alpha f), (f, \alpha f)}}{\arg\max} V_S\left(C, q\right)$$

- A subset of candidate strategies can be eliminated based on ordering of critical values, f and αf

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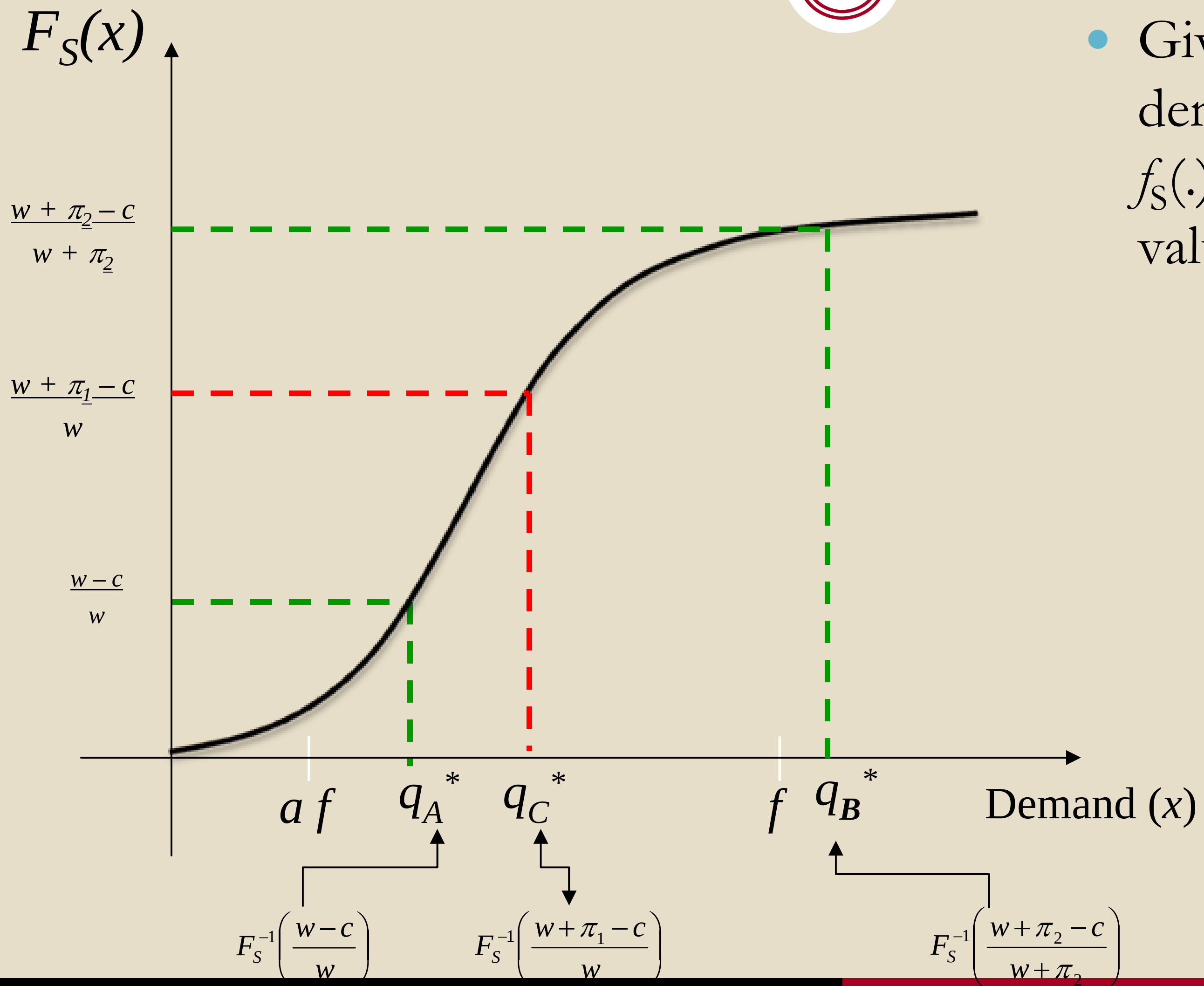
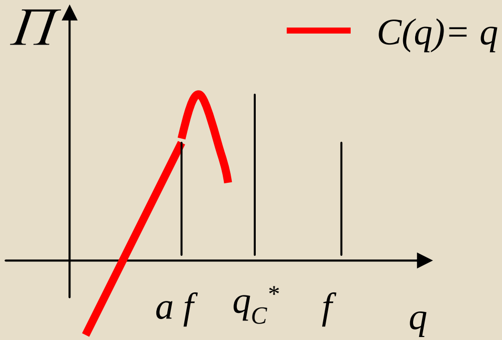
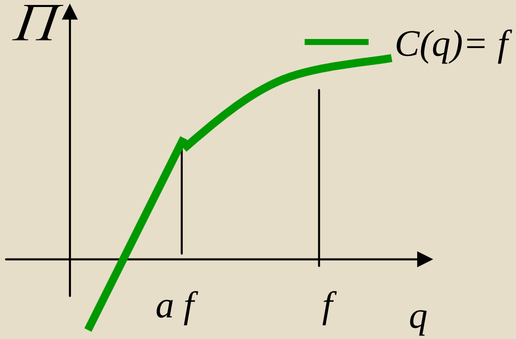
Ordering of Critical Values



- Given f, and choice of demand distribution, $f_S(.)$, ordering of critical values is set.





Candidate Strategies for Consistent Ordering Pairs

Orderings Under		$C(q) = f$			
		1.	**2.**	**3.**	**4.**
$C(q) = q$	*A.*	(f, q_A^*)	$--$	$--$	$--$
	B.	$--$	$(q_C^*, q_C^*), (f, q_B^*)$	$(q_C^*, q_C^*), (f, \alpha f)$	$(q_C^*, q_C^*), (f, f)$
	C.	$--$	$(\alpha f, \alpha f), (f, q_B^*)$	$(\alpha f, \alpha f), (f, \alpha f)$	$(\alpha f, \alpha f), (f, f)$
	D.	$--$	$(f, f), (f, q_B^*)$	$(f, f), (f, \alpha f)$	$(f, f), (f, f)$

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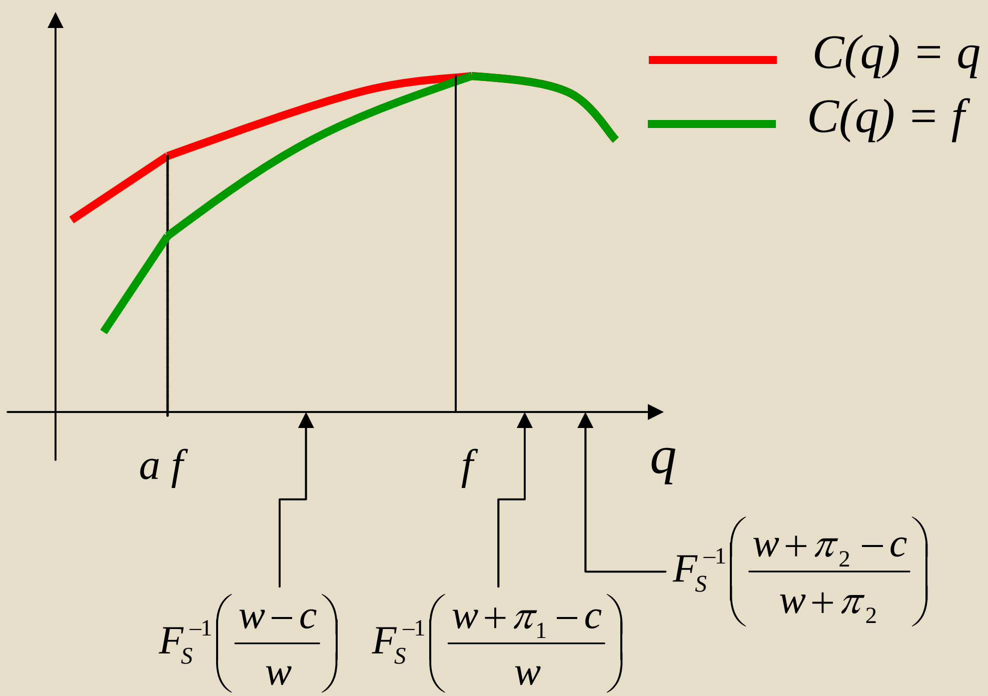
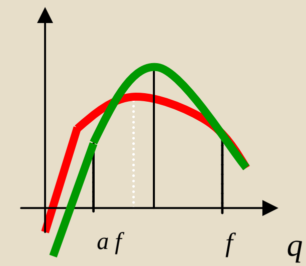
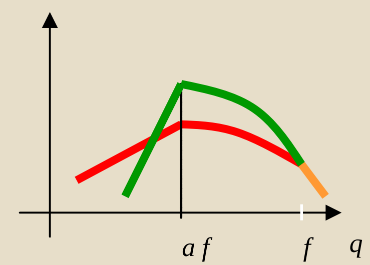
Critical Values Ordering & Optimal Policy

- Given Ordering of Critical Values
  - Optimal Production Quantity known under Commitment Strategies
    - Eight Possible Cases







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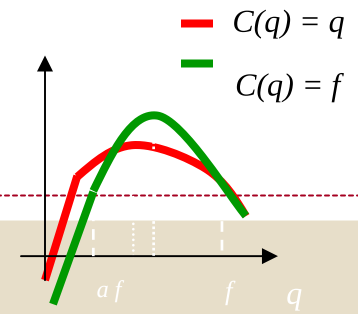
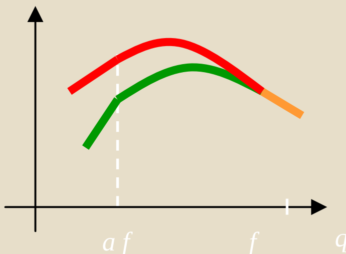
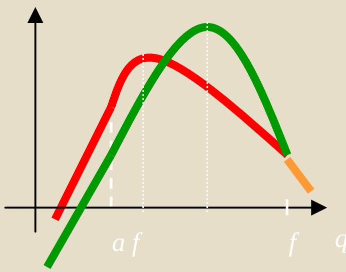
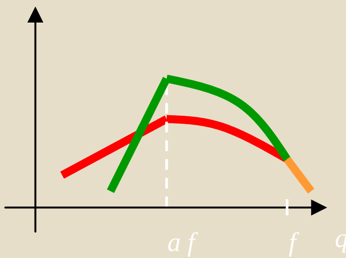
Summary of Analytical Results

- Optimal Commitment Strategy
  - Commitment quantity, $C^*(q)$, non-decreasing in q
  - Supplier incurs either type 1 or type 2 penalty, never both
  - Either Dominant Strategy or Threshold Policy
- Optimal Value Function
  - Sufficient Conditions for Unimodality
- Optimal Strategy
  - Given forecast, compute Ordering of Critical Values
  - Optimal Production Quantity given for Each Strategy
    - Choice of optimal pair based on trade-offs









The Multi-product Problem

- Distinct products
- No substitution
- Forecasts and orders generated independently
- Formulation:

$$\max_{C_i,\, q_i} \sum_{i=1}^{N} V_s \left(q_i,\ C_i,\ \overline{f_i} \right)$$

Subject to:

$$\sum_{i=1}^{N} \mu_i q_i \leq \text{Capacity}$$

$$q_i \geq 0, \quad i = 1, 2, \ldots, N$$

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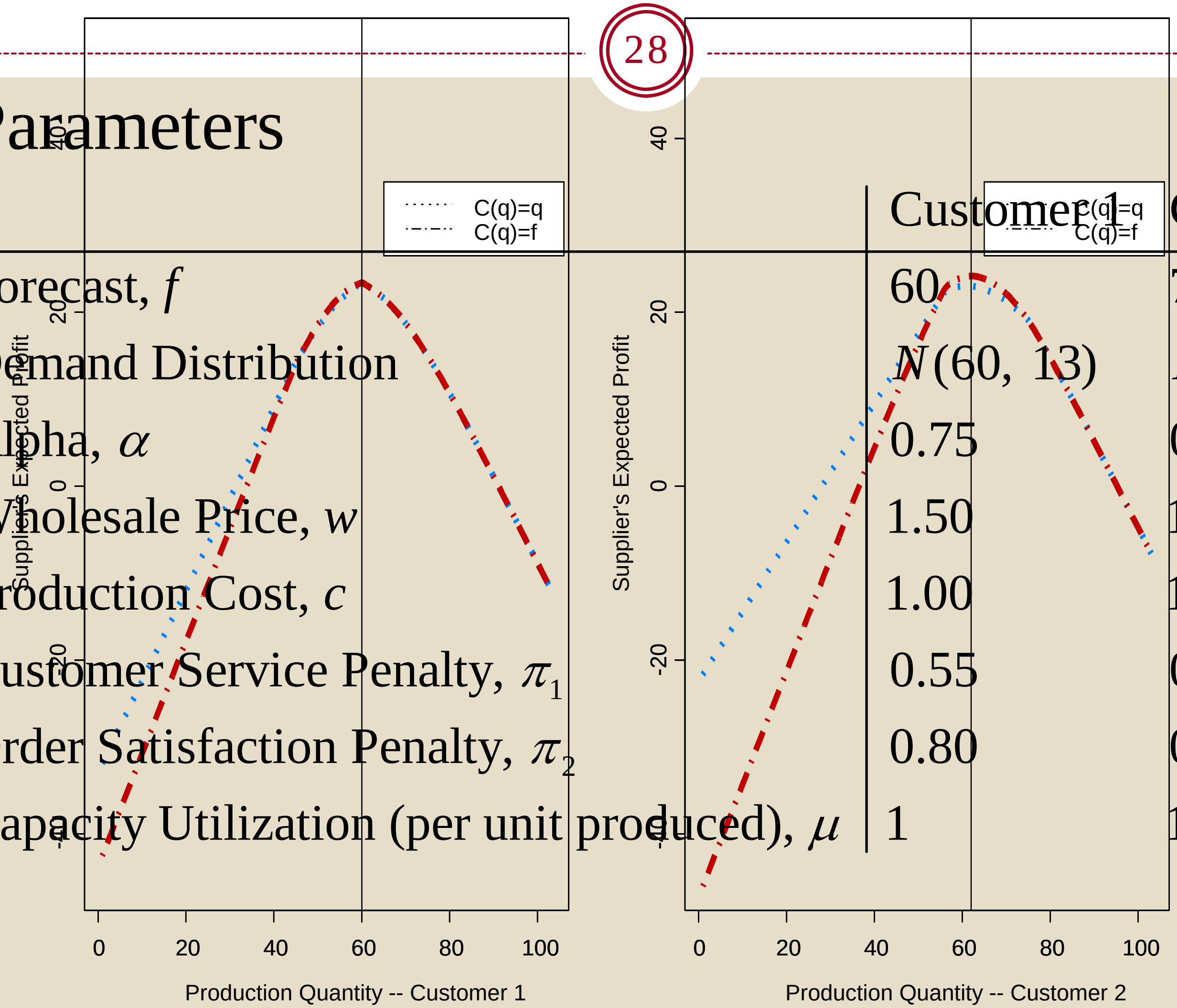
Multi-product Problem: Optimal Policy

- Case 1: Capacity is not binding
  - Problem decomposes into single product problems
- Case 2: Capacity is binding -- A mess!!
  - Value function becomes highly irregular (multimodal)
  - Must consider all possible capacity & allocations

An Example

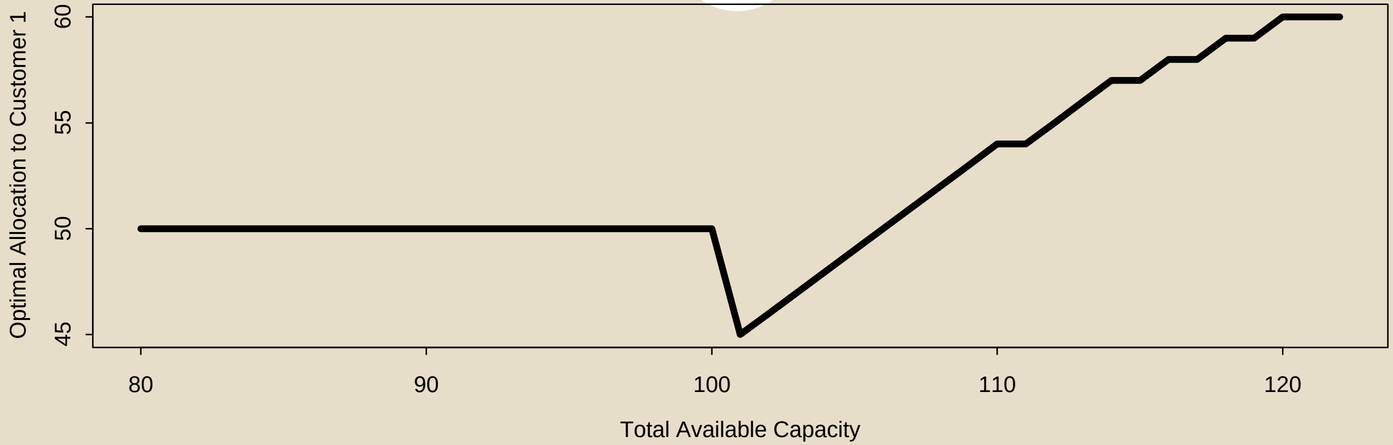
Parameters

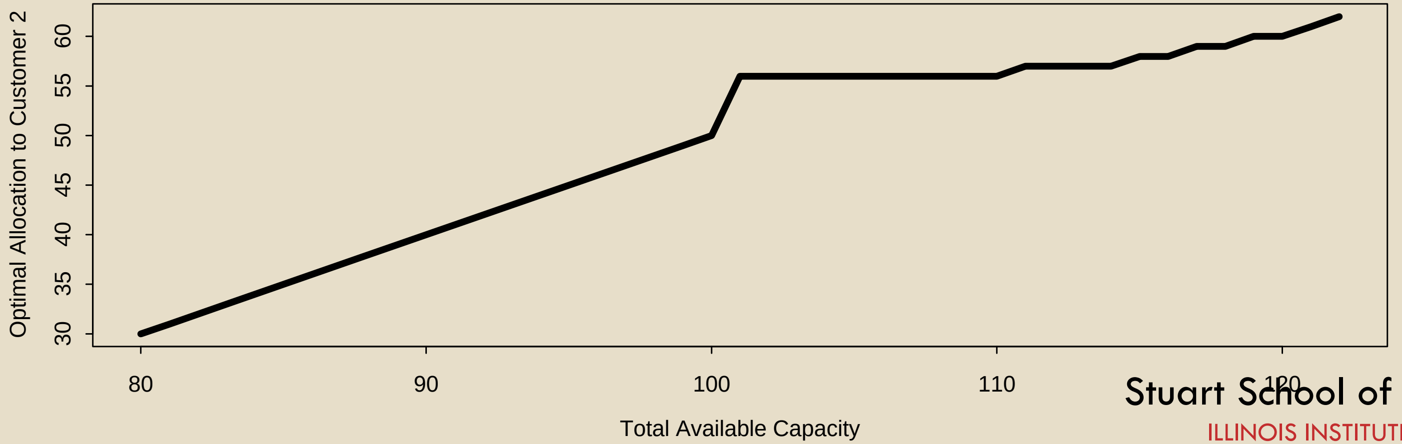
	Customer 1	Customer 2
Forecast, f	60	75
Demand Distribution	$N(60,\ 13)$	$N(60,\ 13)$
Alpha, α	0.75	0.75
Wholesale Price, w	1.50	1.50
Production Cost, c	1.00	1.00
Customer Service Penalty, π_1	0.55	0.30
Order Satisfaction Penalty, π_2	0.80	0.80
Capacity Utilization (per unit produced), μ	1	1



- Unconstrained Solution: $C^*=f,\ q_1^*=60,\ q_2^*=62$

Effect of Capacity Constraint on Productive Capacity allocated to Customers





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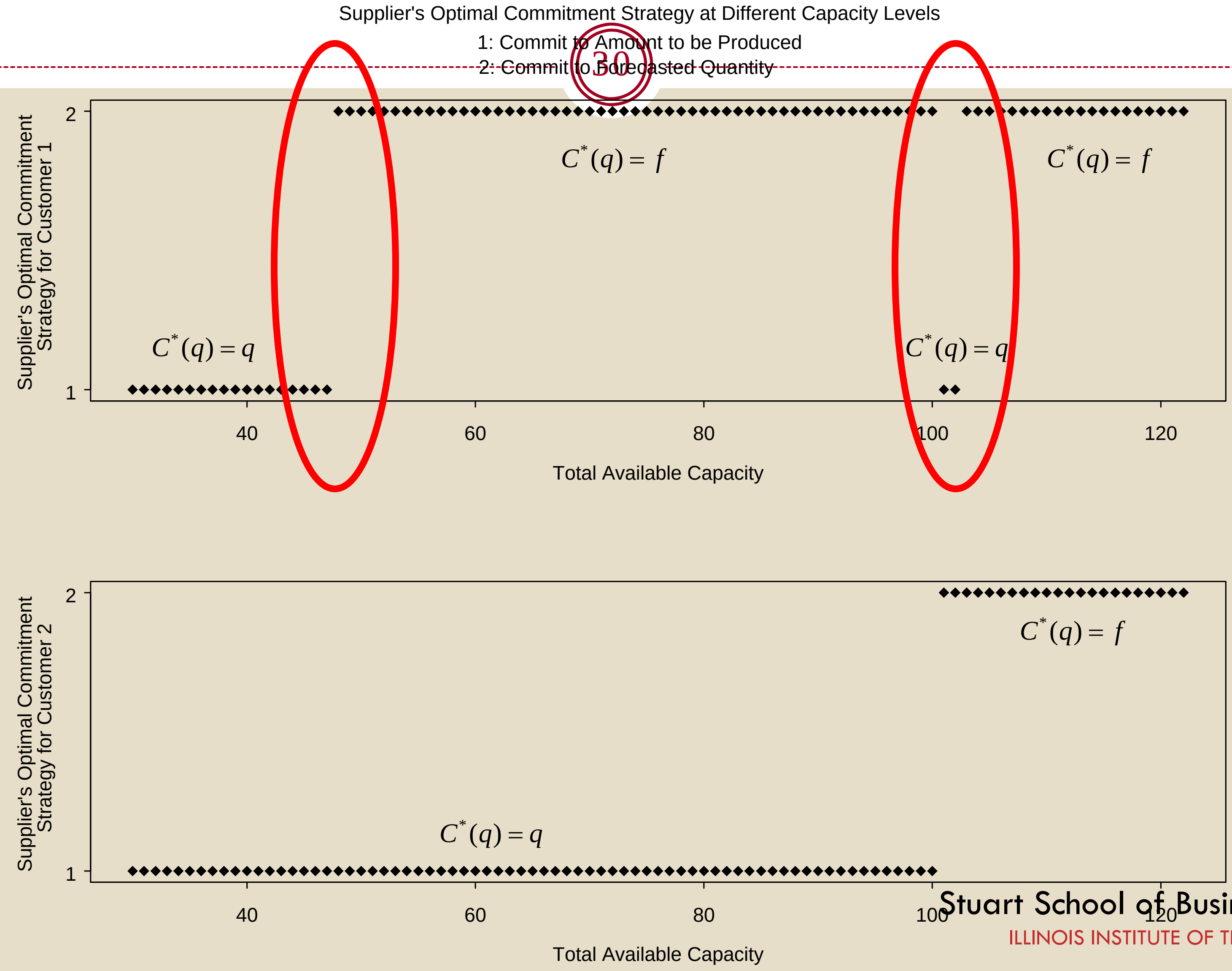
Effect of Capacity on Commitment Strategy



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Effect of Capacity on Allocations and Profits

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Capacitated Problem —Results

- Optimal Capacity Allocation to Customers Not Monotonic in Total Available Capacity

- May Not be Optimal to Allocate Entire Capacity, even if Union of Unconstrained Solutions Exceeds Capacity Limit

Value of Forecast-Commitment Contracts

- Contract Curbs Supplier's Motivation to Underproduce
  - No Contract: $q^{*} = F^{-1}\left(\frac{w-c}{w}\right)$
  - Less than Optimal Production amount(s) in the Presence of the Contract
- Contract Limits Customer's Incentive to Over-forecast
- Provides a Means for Customer to Plausibly Pass High Forecasts to Supplier

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Current and Future Research Directions

- Related Research:
  - Customer's Forecasting Problem
    - Sequential Game, with Customer as First Mover
    - Assume Common Beliefs about Demand and Common Information about Costs
    - Coordination in high number of instances
  - FC-Contract with Strategic Customer (under review)
    - Captures customer incentive to lie in order to receive the delivery penalty

- Other Research Interest
  - Intersection of Operations and Marketing
    - Store-brands vs. National Brands Capacity Allocation
  - Supply Chain Management with Strategic Consumers
    - Conspicuous Consumption (Effect of Snobs & Followers)
    - Forward-Looking Consumers