Supplier Commitment & Production Decisions Under a Forecast-Commitment Forecast

Elizabeth J. Durango-Cohen, Ph. D.

Assistant Professor of Operations Management
Stuart School of Business
Illinois Institute of Technology

(Joint work with Candace Yano)
Overview

- Background & Problem Motivation
- Taxonomy of Supply Contracts Literature
- Operational Model of Production Planning with Commitments under a Forecast-Commitment Contract
  - Single Product, Single Customer Problem
    - Model Formulation
    - Results & Managerial Insights
    - Effect of a Capacity Constraint
- Related Research
  - Customer Problem of Forecast Selection
- Concluding Remarks
Background: Supply Chain Management

- **What is a Supply Chain?**
  - A network of facilities that perform the functions of:
    - Procurement of Materials
    - Transformation of Materials
    - Distribution of Finished Goods to Customers

- **Supply Chain Management**
  - Logistics of Controlling Material and Information flows
Example of a Supply Chain in High-technology Industry
Problem Motivation

- Worked with Application Specific Integrated Circuits (ASICs) Manufacturer
  - ICs embedded in other divisions’ products

- Originally: Internal supplier and cost center
  - Capacity allocation strategy set at corporate level

- At the time: Profit center with internal customers
  - Allocation based on maximizing division’s profit (myopic)
  - Transitioning: Wholly-owned Subsidiary; mix of internal and external customers (customer relationship management!)

- Conflicting Short-term and Long-term Goals
Problem Motivation (cont.)

- ASICs market is extremely volatile — To gain a competitive advantage
  - Manufacturer willing to make commitments against 1-period-ahead forecasts (viewed as strategic weapon)
  - Alternative to point-system for prioritizing customers under development

- Contracts are negotiated as new product generations are introduced. Contracts specify:
  - Prices, production, holding and penalty costs

- Research Goal: Develop commitment and capacity allocation models that incorporate existing business practices
Contract Structure

- **One period in advance:**
  - Customer provides order forecast, $\overline{f}$
  - Supplier makes a commitment to the customer based on forecast, $C$
  - Supplier decides on a production quantity, $q$

- **When orders are placed:**
  - Customer bound to order a fraction of forecast, $\alpha$, unless supplier committed to a lesser value, $d = \min(\alpha \overline{f}, C)$
  
  Let $\times \equiv$ demand
  
  Customer takes delivery of
  
  $$\text{Delivery Amt.} = \min(q, \max(\times, d))$$
Contract Structure: Penalty Scheme

- **Commitment Penalty**: for failing to commit up to the forecast
  \[ \pi_1 \cdot (\text{Forecasted Amount} - \text{Commitment})^+ \]

- **Delivery Penalty**: for not delivering on commitment amount
  \[ \pi_2 \cdot (\min (\text{Demand}, \text{Committed Amount}) - \text{Units Supplied})^+ \]
Supplier’s Problem

Goal: choose commitment and production quantity that maximize profits
Taxonomy – Brief Overview

- Different Contract Types:
  - Buy-Back Contracts (Pasternack 1985)
    - Full price returns for partial order, or partial refund for all returns
  - Pay-to-Delay (Brown and Lee 1999)
    - Fixed-fee upfront
  - Contracts with Options (Barnes-Schuster et al. (2002))
    - Apparel industry – Two period Model
  - Revenue-Sharing Contracts (Lariviere and Cachon (2005))
    - Video industry – Supply Chain Coordination
  - Push, Pull and Advance Purchase Contracts (Cachon 2004)
    - Push (Price-only), Pull (Vendor-Managed Inventory) – Allocation of Inventory Risk
  - Quantity Flexibility (Tsay 1999)
    - Customer agrees to purchase a fraction of forecast
    - Supplier agrees to supply up to a fraction above
Supplier’s Problem

Goal: choose commitment and production quantity that maximize profits
Notation

\[ w \] Wholesale price
\[ c \] Variable production
\[ \alpha \] Fraction of forecast that customer is obligated to purchase
\[ d \] Minimum required order quantity
\[ f \] Customer forecast provided to supplier
\[ x \] R.V. for the customer demand in the current period
\[ f_s(x | f) \] Conditional density of customer demand given forecast \((f)\)

Decision Variables:
\[ q \] Production quantity
\[ C \] Amount committed to the customer

\[ \Pi_s(q, C, f) \] is the supplier's profit function given customer's forecast
Supplier’s Profit Maximization Problem:

\[ \Pi_s(q,C,f) = \max_{C,q} \left\{ \begin{array}{l}
\text{Revenue} \\
\quad w \left[ \int_{x=0}^{d} df_s(x)dx + \int_{x=d}^{q} xf_s(x)dx + \int_{d=q}^{\infty} qf_s(x)dx \right] \\
- cq \\
- \pi_1 [f - C]^+ \\
- \pi_2 \left[ \int_{x=q}^{C} (x-q)f_s(x)dx + \int_{x=C}^{\infty} (C-q)f_s(x)dx \right] \\
\end{array} \right\} \]

Production Cost
Commitment Penalty
Customer Satisfaction/Delivery Penalty
Solution Approach

- Want \( C \) and \( q \) that maximizes \( \Pi_S(q, C, f) \)

1. For all production quantity values, \( q \), we find the optimal commitment value
   
   - \( C^*(q) = \arg \max_C [\Pi_S(q, C, f)] \)

2. Given \( C^*(q) \), we find \( q \) that maximizes \( \Pi_S(q, C^*(q), f) \).
Optimal Commitment Response — Results

- **Theorem:** An optimal supplier commitment response, $C^*(q)$, for a given production quantity, $q$, and the customer forecast, $f$, is

  $$C^*(q) = q \text{ or } \bar{f}.$$  

That is, the supplier will either commit to the amount to be produced or commit to the amount forecasted by the customer.

- **Lemma:** The supplier's optimal commitment, $C^*(q)$, is non-decreasing in the amount to be produced, $q$. 
Find Optimal Production Quantity, given Commitment Response, \( \max_q \Pi_s(q, C(q), f) \).

- When \( C(q) = q \). For \( \alpha f \leq q < f \) the value function is

\[
\Pi(q, C(q) = q, f) = -cq + w \left[ \int_{x=0}^{\alpha f} (\alpha f) f_s(x) dx + \int_{x=\alpha f}^{q} xf_s(x) dx + \int_{x=q}^{\infty} qf_s(x) dx \right] - \pi_1[f - q]
\]

- The first and second derivatives are:

\[
\frac{\partial \Pi(q, q, f)}{\partial q} = -c + w[1 - F_s(q)] + \pi_1
\]

\[
\frac{\partial^2 \Pi(q, q, f)}{\partial q^2} = -wf_s(q) \leq 0
\]

- with stationary point

\[
\hat{q} = q^*_C = F_s\left(\frac{w + \pi_1 - c}{w}\right)
\]

- So \( \Pi_s(q, q, f) \) can be concave, concave increasing or decreasing in \( q \).
Finding $q^*$ under $C(q) = q$ (cont.)

- For $f \leq q$ the value function is
  \[
  \Pi(q, q, f) = -cq + w\left[\int_{x=0}^{\alpha f} (\alpha f) f_s(x) dx + \int_{x=\alpha f}^{q} xf_s(x) dx + \int_{x=q}^{\infty} q f_s(x) dx\right]
  \]

- The first and second derivatives are:
  \[
  \frac{\partial \Pi(q, q, f)}{\partial q} = -c + w[1 - F_S(q)]
  \]
  \[
  \frac{\partial^2 \Pi(q, q, f)}{\partial q^2} = -wf_S(q) \leq 0
  \]

- with stationary point
  \[
  \hat{q} = q_A^* = F_S^{-1}\left(\frac{w-c}{w}\right)
  \]

- So $\Pi_S(q, q, f)$ can be concave, or concave decreasing in $q$.
Forms of Value Function, $C(q) = q$

$q^* = F_s^{-1}\left(\frac{w-c}{w}\right)$

$q^* = \alpha f$

$q^* = f$

$q^* = F_s^{-1}\left(\frac{w+\pi_1-c}{w}\right)$

$q^* = F_s^{-1}\left(\frac{w-c}{w}\right)$
Finding $q^*$ under strategy $C(q) = f$

- If the supplier’s commitment strategy is to commit to the forecasted amount, the optimal production quantities are:

$$ q^* = \begin{cases} 
\alpha f & \text{or } f \\
F^{-1}_S \left( \frac{w + \pi_2 - c}{w + \pi_2} \right) & \alpha f \leq q < f \\
F^{-1}_S \left( \frac{w - c}{w} \right) & f \leq q
\end{cases} $$
Related Problems

- **Problem A:** Supplier not liable for any shortages
  \[ q_A^* = F_s^{-1} \left( \frac{w - c}{w} \right) \]

- **Problem B:** Supplier is fully liable for any shortages
  \[ q_B^* = F_s^{-1} \left( \frac{w + \pi_2 - c}{w + \pi_2} \right) \]

- **Problem C:** Supplier liable only for commitment amount
  \[ q_C^* = F_s^{-1} \left( \frac{w + \pi_1 - c}{w} \right) \]

Optimal Policy Depends on these Critical Values
**Theorem:** The optimal commitment and production quantity pair for the supplier is:

\[
(C^*, q^*) = \arg\max \quad V_S (C, q)
\]

\[
\left( f, q_A^* \right), \left( f, q_B^* \right), \left( q_B^*, q_C^* \right), \left( f, q_C^* \right), \left( f, f \right), \left( \alpha f, \alpha f \right), \left( f, \alpha f \right)
\]

A subset of candidate strategies can be eliminated based on ordering of critical values, \( f \) and \( \alpha f \).
Ordering of Critical Values

Given $f$, and choice of demand distribution, $f_S(.)$, ordering of critical values is set.

- $C(q) = q$
- $C(q) = f$

$F_S(x)$

$w + \pi_2 - c$

$w + \pi_2$

$w - c$

$w$

$af$

$q_A^*$

$q_C^*$

$f$

$q_B^*$

Demand $(x)$

$\Pi$

$\Pi$
## Candidate Strategies for Consistent Ordering Pairs

<table>
<thead>
<tr>
<th>Orderings Under $C(q) = q$</th>
<th>1.</th>
<th>2.</th>
<th>3.</th>
<th>4.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$(f, q_A^*)$</td>
<td>--</td>
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<td>--</td>
</tr>
<tr>
<td>B</td>
<td>--</td>
<td>$(q_C^<em>, q_C^</em>)$, $(f, q_B^*)$</td>
<td>$(q_C^<em>, q_C^</em>)$, $(f, \alpha f)$</td>
<td>$(q_C^<em>, q_C^</em>)$, $(f, f)$</td>
</tr>
<tr>
<td>C</td>
<td>--</td>
<td>$(\alpha f, \alpha f)$, $(f, q_B^*)$</td>
<td>$(\alpha f, \alpha f)$, $(f, \alpha f)$</td>
<td>$(\alpha f, \alpha f)$, $(f, f)$</td>
</tr>
<tr>
<td>D</td>
<td>--</td>
<td>$(f, f)$, $(f, q_B^*)$</td>
<td>$(f, f)$, $(f, \alpha f)$</td>
<td>$(f, f)$, $(f, f)$</td>
</tr>
</tbody>
</table>
Given Ordering of Critical Values

- Optimal Production Quantity known under Commitment Strategies
  - Eight Possible Cases

\[ C(q) = q \]
\[ C(q) = f \]

\[ F_S^{-1}\left( \frac{w-c}{w} \right) \]
\[ F_S^{-1}\left( \frac{w+\pi_1-c}{w} \right) \]
\[ F_S^{-1}\left( \frac{w+\pi_2-c}{w+\pi_2} \right) \]
Summary of Analytical Results

- Optimal Commitment Strategy
  - Commitment quantity, $C^*(q)$, non-decreasing in $q$
  - Supplier incurs either type 1 or type 2 penalty, never both
  - Either Dominant Strategy or Threshold Policy

- Optimal Value Function
  - Sufficient Conditions for Unimodality

- Optimal Strategy
  - Given forecast, compute Ordering of Critical Values
  - Optimal Production Quantity given for Each Strategy
    - Choice of optimal pair based on trade-offs
The Multi-product Problem

- Distinct products
- No substitution
- Forecasts and orders generated independently
- Formulation:

\[
\max_{C_i, q_i} \sum_{i=1}^{N} V_s \left( q_i, C_i, \bar{f}_i \right)
\]

Subject to:

\[
\sum_{i=1}^{N} \mu_i q_i \leq \text{Capacity}
\]

\[
q_i \geq 0, \quad i = 1, 2, \ldots, N
\]
Multi-product Problem: Optimal Policy

- Case 1: Capacity is not binding
  - Problem decomposes into single product problems

- Case 2: Capacity is binding -- A mess!!
  - Value function becomes highly irregular (multimodal)
  - Must consider all possible capacity & allocations
An Example

Optimal Customer Forecasts in FC-Contracts

Durango-Cohen

Stuart School of Business
ILLINOIS INSTITUTE OF TECHNOLOGY

Parameters

Forecast, $f$
Demand Distribution
Alpha, $\alpha$
Wholesale Price, $w$
Production Cost, $c$
Customer Service Penalty, $\pi_1$
Order Satisfaction Penalty, $\pi_2$
Capacity Utilization (per unit produced), $\mu$

Customer 1
Forecast, $f$ = 60
Demand Distribution $N(60, 13)$
Alpha, $\alpha$ = 0.75
Wholesale Price, $w$ = 1.50
Production Cost, $c$ = 1.00
Customer Service Penalty, $\pi_1$ = 0.55
Order Satisfaction Penalty, $\pi_2$ = 0.80
Capacity Utilization (per unit produced), $\mu$ = 1

Customer 2
Forecast, $f$ = 75
Demand Distribution $N(60, 13)$
Alpha, $\alpha$ = 0.75
Wholesale Price, $w$ = 1.50
Production Cost, $c$ = 1.00
Customer Service Penalty, $\pi_1$ = 0.30
Order Satisfaction Penalty, $\pi_2$ = 0.80
Capacity Utilization (per unit produced), $\mu$ = 1

Unconstrained Solution: $C^* = f$, $q_1^* = 60$, $q_2^* = 62$

An Example
Effect of Capacity Constraint on Productive Capacity allocated to Customers
Effect of Capacity on Commitment Strategy

Supplier's Optimal Commitment Strategy at Different Capacity Levels
1: Commit to Amount to be Produced
2: Commit to Forecasted Quantity

$C^*(q) = q$

$C^*(q) = f$

Total Available Capacity

Optimal Customer Forecasts in FC-Contracts  Durango-Cohen
Effect of Capacity on Allocations and Profits

Total Available Capacity = 100

Total Available Capacity = 101
Capacitated Problem — Results

- Optimal Capacity Allocation to Customers Not Monotonic in Total Available Capacity

- May Not be Optimal to Allocate Entire Capacity, even if Union of Unconstrained Solutions Exceeds Capacity Limit
Value of Forecast-Commitment Contracts

- Contract Curbs Supplier's Motivation to Underproduce
  - No Contract: \( q^* = F^{-1}\left(\frac{w-c}{w}\right) \)
  - Less than Optimal Production amount(s) in the Presence of the Contract

- Contract Limits Customer's Incentive to Over-forecast

- Provides a Means for Customer to Plausibly Pass High Forecasts to Supplier
Current and Future Research Directions

- Related Research:
  - Customer’s Forecasting Problem
    - Sequential Game, with Customer as First Mover
    - Assume Common Beliefs about Demand and Common Information about Costs
    - Coordination in high number of instances
  - FC-Contract with Strategic Customer (under review)
  - Captures customer incentive to lie in order to receive the delivery penalty

- Other Research Interest
  - Intersection of Operations and Marketing
    - Store-brands vs. National Brands Capacity Allocation
  - Supply Chain Management with Strategic Consumers
    - Conspicuous Consumption (Effect of Snobs & Followers)
    - Forward-Looking Consumers