OPTIMIZATION MODELING FOR TRADEOFF ANALYSIS OF HIGHWAY INVESTMENT ALTERNATIVES

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- Bridge Management Systems
- Maintenance Management Systems
- Safety Management Systems
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- Increasing System Demand
- Budget Pressure
- Accountability Requirements
- Technological Advancements
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Updated System → List of Candidate Projects from Needs Assessment

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- Project Selection

Project Implementation → Budget
Optimization Formulation for Systemwide Highway Project Selection

- As the 0-1 Multi-Choice Multidimensional Knapsack Problem
  - Multi-choice corresponds to multiple categories of budgets designated for different highway management programs
  - Multi-dimension refers to a multiyear project implementation period, and
  - The objective is to select a subset from all economically feasible candidate projects to achieve maximized total benefits under various constraints.

- Basic Model

Maximize $A^T.X$

Subject to $C_{kt}^T.X \leq B_{kt}$

where $A$ is the vector of benefits of $N$ projects, $A = [a_1, a_2, ..., a_N]^T$, $X$ is the decision vector for all decision variables, $X= [x_1, x_2, ..., x_N]^T$, $C_{kt}$ is the vector of costs of $N$ projects using budget from management program $k$ in year $t$, and $B_{kt}$ is the budget available for management program $k$ in year $t$. 


Addressing Budget Uncertainty in Project Selection

Issues of Budget Uncertainty in Project Selection

Using Recourse Decisions to Address Budget Uncertainty

<table>
<thead>
<tr>
<th>Year</th>
<th>1 to $t_1$</th>
<th>$t_1$ to $t_2$</th>
<th>...</th>
<th>$t_{(L-2)}$ to $t_{(L-1)}$</th>
<th>$t_{(L-1)}$ to $t_L$</th>
<th>$t_L$ to $t_{(L+1)}$</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Budget</td>
<td>1 possibility</td>
<td>$s_2$ possibilities</td>
<td>...</td>
<td>$s_{(L-1)}$ possibilities</td>
<td>$s_L$ possibilities</td>
<td>$s_{(L+1)}$ possibilities</td>
<td>...</td>
</tr>
</tbody>
</table>

Stage 1: Deterministic (Initially estimated budgets)

Stage 2: Deterministic  Stochastic ($p_2 = s_2.s_3...s_{L-1}.s_L.s_{L+1}...$ combinations)

... 

Stage $L$-1: Deterministic  Stochastic ($p_{L-1} = s_{L-1}.s_L.s_{L+1}...$ combinations)

Stage $L$: Deterministic  Stochastic ($p_L = s_L.s_{L+1}...$ combinations)

Stage $L$+1: Deterministic  Stochastic ($p_{L+1} = s_{L+1}...$ combinations)
A Stochastic Model with Ω-stage Budget Recourse Decisions

Maximize $A^T . X_1 + E_{\xi_2}[Q_2(X_2(p), \xi_2)] + \ldots + E_{\xi_\Omega}[Q_\Omega(X_\Omega(p), \xi_\Omega)]$ \hfill (1)

Stage 1:
Subject to $C_{kt}^T . X_1 \leq E(B_{kt}^1)$ \hfill (2)

Stage 2:
$E_{\xi_2}[Q_2(X_2(p), \xi_2)] = \max \left\{ A^T . X_2(p) \mid B_{kt}^1(p) = E(B_{kt}^1) \right\}$ \hfill (3)
Subject to $C_{kt}^T . X_2(p) \leq B_{kt}^2(p)$ \hfill (4)
$X_1 + X_2(p) \leq 1$ \hfill (5)

Stage L:
$E_{\xi_L}[Q_L(X_L(p), \xi_L)] = \max \left\{ A^T . X_L(p) \mid B_{kt}^L(p) = E(B_{kt}^L) \right\}$ \hfill (6)
Subject to $C_{kt}^T . X_L(p) \leq B_{kt}^L(p)$ \hfill (7)
$X_1 + X_2(p) + \ldots + X_L(p) \leq 1$ \hfill (8)

Stage Ω:
$E_{\xi_\Omega}[Q_\Omega(X_\Omega(p), \xi_\Omega)] = \max \left\{ A^T . X_\Omega(p) \mid B_{kt}^\Omega(p) = E(B_{kt}^\Omega) \right\}$ \hfill (9)
Subject to $C_{kt}^T . X_\Omega(p) \leq B_{kt}^\Omega(p)$ \hfill (10)

where $A$ is the vector of benefits of $N$ projects, $A = [a_1, a_2, \ldots, a_N]^T$, $C_{kt}$ is the vector of costs of $N$ projects using budget from management program $k$ in year $t$, $C_{kt} = [c_{1kt}, c_{2kt}, \ldots, c_{Nkt}]^T$, $X_L(p)$ is the decision vector using budget $B_{kt}^L(p)$ at stage $L$, $X_L(p) = [x_1, x_2, \ldots, x_N]^T$, $a_i$ is benefits of project $i$, $i = 1, 2, \ldots, N$, $c_{ikt}$ is costs of project $i$ using budgets from management program $k$ in year $t$, $x_i$ is the decision variable for project $i$, $\xi_L$ is randomness associated with budgets at stage $L$ and decision space, $Q(X_L(p), \xi_L)$ is the recourse function at stage $L$, $E_{\xi_L}[Q(X_L(p), \xi_L)]$ is the mathematical expectation of the recourse function at stage $L$, $B_{kt}^L(p)$ is the $p^{th}$ possibility of budget for management program $k$ in year $t$ at stage $L$, $p(B_{kt}^L(p))$ is the probability of having budget scenario $B_{kt}^L(p)$. 
Budget for **Stage L** Computation

- **Criterion to determine Budget for Stage L Computation**

  - For yearly constrained budget scenario: Minimize $\Delta B_L(p) = \sum_{k=1}^{K} \sum_{t=1}^{M} \left[ (B_{kt}(p) - E(B_{kt}))^2 \right]$ where $E(B_{kt}) = \sum_{t=1}^{M} p \cdot E(B_{kt}(p))$.

  - For cumulative budget scenario: Minimize $\Delta B_L(p) = \sum_{t=1}^{M} \left[ \sum_{p=1}^{K} E(B_{kt}(p)) \cdot E(B_{kt}(p)) \right]$

### An Example

#### Budget Possibility 1 (10% Chance)

<table>
<thead>
<tr>
<th>$k$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
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<td>25% Lower</td>
<td></td>
<td></td>
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<tr>
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<td>100</td>
<td>120</td>
<td>75</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>100</td>
<td>1.2</td>
<td>0.75</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>100</td>
<td>120</td>
<td>75</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1.2</td>
<td>75</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>100</td>
<td>120</td>
<td>75</td>
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</table>

#### Budget Possibility 2 (25% Chance)

<table>
<thead>
<tr>
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<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>No change</td>
<td>No change</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
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</tr>
<tr>
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<td>120</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1.2</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
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<td>100</td>
<td>120</td>
<td>100</td>
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</tbody>
</table>

#### Budget Possibility 3 (65% Chance)

<table>
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<tr>
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<th>3</th>
<th>4</th>
</tr>
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<tbody>
<tr>
<td>$t$</td>
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<td>25% Higher</td>
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</tr>
<tr>
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<td>100</td>
<td>120</td>
<td>125</td>
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</tr>
<tr>
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<td>100</td>
<td>100</td>
<td>1.25</td>
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<td>1</td>
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<td>125</td>
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<td>1.25</td>
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<tr>
<td>5</td>
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<td>100</td>
<td>125</td>
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</table>

#### Expected Budget

<table>
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<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
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<td>Expected Budget</td>
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<td></td>
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<td>100</td>
<td>120</td>
<td>114</td>
<td></td>
</tr>
<tr>
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<td>100</td>
<td>100</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>100</td>
<td>114</td>
<td></td>
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<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>114</td>
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</tr>
<tr>
<td>5</td>
<td>100</td>
<td>100</td>
<td>114</td>
<td></td>
</tr>
</tbody>
</table>
Enhanced Stochastic Model

- **Incorporate Segment-Based Project Implementation Option**
  - Tie-ins of multiple projects within one highway segment or across multiple highway segments for actual implementation
  - Benefits of all constituent projects of a segment-based “project group” added together
  - The constituent projects may request budgets from different programs in multiple years
  - The size of the decision vector in the stochastic model is reduced.

- **Incorporate Corridor-Based Project Implementation Option**
  - As an extension of segment-based project implementation option, the tie-ins of multiple projects within one or more highway segments is extended to a freeway corridor or a major urban arterial corridor
  - Benefits of all constituent projects of a corridor-based “grand project group” combined
  - The constituent projects may request budgets from different programs in multiple years
  - The size of the decision vector in the stochastic model is further reduced.

- **Incorporate Deferment-Based Project Implementation Option**
  - Some large-scale projects may have a high risk of being deferred due to right-of-way acquisition delays, design changes, and significant environmental impacts, etc.
  - Project benefits and costs adjusted according to the number of years of deferment
  - The size of the decision vector in the stochastic model remains unchanged.
Theorem of Lagrange Multipliers

- Redefined the optimization model for Stage L
  Objective \( \text{Max} \quad z(Y_L) = A^T Y_L \)
  Subject to \( C_{kt}^T Y_L \leq B_{kt}^L \), where \( Y_L \) is stage \( L \) decision vector with 0/1 integer elements.

- For non-negative Lagrange Multipliers \( \lambda_{kt} \), the Lagrangian relaxation of the model can be written as
  Objective \( z_{LR}(\lambda_{kt}) = \max \)
  Subject to \( Y_L \) with 0/1 integer elements.

- The unconstrained solution to \( z'_{LR}(\lambda_{kt}) = \max \) is

- The solution algorithm for the original optimization model needs to focus on determining Lagrange Multipliers \( \lambda_{kt} \) to satisfy the following conditions:
  \[ \sum_{k=1}^{K} \sum_{t=1}^{M} \lambda_{kt} \left( B_{kt}^L - C_{kt}^T Y_L \right) = 0 \]
  \[ Y_L^* = \begin{cases} 
  1, & \text{if } A^T \sum_{k=1}^{K} \sum_{t=1}^{M} \lambda_{kt} C_{kt}^L > 0 \\
  0, & \text{otherwise}
  \end{cases} \]
Proposed Algorithm for *Stage L* Computations

- **Step 0 (Initialization and Normalize)**
  - Determine budget $B_{kt}^L(p)$ for Stage $L$ such that $\Delta B^L(p) = \min \{ B_{kt}^L(1), B_{kt}^L(2), ..., B_{kt}^L(p_L) \}$
  - Select all projects and bore projects by benefits ($A_i$) in descending order
  - Normalize contract costs and budget for each $(k, t)$:

- **Step 1 (Determine the Most Violated Constraint $k, t$)**
  - Set $C'_{kt} = \max \{ C_{kt} \}$ for all $k, t$

- **Step 2 (Compute the Increase of Lagrange Multiplier Value $\lambda_{kt}$)**

- **Step 3 (Increase $\lambda_{kt}$ by and Reset $X_i$ the Value Zero)**
  - Let $C_{kt} = C_{kt} - c'_{ikt}$ for all $k, t$
  - Remove project $i$ and reset decision variable $x_i = 0$
  - If $C_{kt} \leq 1$ for all $k, t$, go to Step 4. Otherwise, go to Step 1.

- **Step 4 (Improve Solution)**
  - Check whether the projects with zero-variable values can have the value one without violating the constraints $C_{kt} \leq 1$. 

$$
\sum_{n=1}^{1} c_{ik} = c_{ik}/B_{kt}^1(p)B_{kt}^2(p) = 1 \text{ and } c_{kt} = \sum_{k=1}^{n} c_{ik}.
$$
Proposed Algorithm for **Stage L** Operations (Con’t)

- **Step 5 (Further Improve Solution with Budget Carryover)**

A small amount of budget might be left after project selection and it could be carried over to the immediate following year one year at a time to repeat Steps 1 to 4 to further improve the solution.

<table>
<thead>
<tr>
<th>Before</th>
<th>( B_{k1}^L(p) )</th>
<th>( B_{k2}^L(p) )</th>
<th>...</th>
<th>( B_{k,t-1}^L(p) )</th>
<th>( B_{kt}^L(p) )</th>
<th>( B_{k,t+1}^L(p) )</th>
<th>...</th>
<th>( B_{kM}^L(p) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>After</td>
<td>0</td>
<td>0</td>
<td>...</td>
<td>0</td>
<td>0</td>
<td>( B_{k,t+1}^L(p) )</td>
<td>...</td>
<td>( B_{kM}^L(p) )</td>
</tr>
</tbody>
</table>

One-period budget carryover for remaining budget from year \( t \) to year \( t+1 \):

Increase budget \( B_{k,t+1}^L(p) \) by \( \Delta B_{kt}^L(p) = B_{kt}^L(p) - C_{kt} \) and this leaves \( B_{kt}^L(p) = 0 \) after budget carryover.

- Hold solution for the preceding years from 1 to \( t \)
- Re-optimize for the remaining years from \( t+1 \) to \( M \)
- Repeat until reaching the last year \( M \).
Computational Complexity of the Proposed Algorithm

- Steps 1-4: Computational complexity is $O(M.N^2)$
- Step 5: Budget carryover requires $M$ iterations
- $\Omega$-Stage recourses needs at most $M$ interactions

This gives an overall complexity of $O(M^3N^2)$. Since $M<<N$, the algorithm remains a complexity of $O(N^2)$. 

 Candidate Project Data - Preparation

Eleven-year data on 7,380 candidate projects proposed for Indiana state highway programming during 1996-2006 were used to apply the proposed heuristic approach for systemwide project selection.

Examples of estimated project-level life-cycle benefits:

<table>
<thead>
<tr>
<th>Project No.</th>
<th>Year</th>
<th>Lanes</th>
<th>Length (Miles)</th>
<th>AADT</th>
<th>Work Type</th>
<th>Project Cost</th>
<th>AC</th>
<th>VOC</th>
<th>Mobility</th>
<th>Safety</th>
<th>Env.</th>
<th>Total Benefits</th>
</tr>
</thead>
<tbody>
<tr>
<td>12021</td>
<td>2000</td>
<td>4</td>
<td>0.11 69,200</td>
<td>Bridge widening</td>
<td>2,291,000</td>
<td>4</td>
<td>22</td>
<td>1</td>
<td>55</td>
<td>19</td>
<td>11,703,264</td>
<td></td>
</tr>
<tr>
<td>12040</td>
<td>2000</td>
<td>4</td>
<td>0.50 32,630</td>
<td>Pavement resurfacing</td>
<td>4,620,000</td>
<td>2</td>
<td>33</td>
<td>1</td>
<td>37</td>
<td>27</td>
<td>6,365,844</td>
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</tr>
<tr>
<td>12077</td>
<td>2000</td>
<td>2</td>
<td>2.06 3,170</td>
<td>Pavement resurfacing</td>
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<td>3</td>
<td>27</td>
<td>1</td>
<td>46</td>
<td>23</td>
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<tr>
<td>12158</td>
<td>1999</td>
<td>2</td>
<td>3.70 16,770</td>
<td>Added travel lanes</td>
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<td>2</td>
<td>30</td>
<td>7</td>
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<tr>
<td>21749</td>
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<td>13.63 4,190</td>
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<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>63,943,225</td>
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<tr>
<td>21825</td>
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<td>2.53 11,150</td>
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<td>10</td>
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<td>31</td>
<td>27</td>
<td>1,505,738</td>
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<td>21931</td>
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<td>Rigid pavement replace</td>
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<td>28</td>
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<td>22</td>
<td>5,702,627</td>
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</table>

... ... ... ... ... ... ... ... ...

Budget Data

The annual average budgets designated for new construction, pavement preservation, bridge preservation, maintenance, safety improvements, roadside improvements, ITS installations, and miscellaneous programs were approximately 700 million dollars with 4 percent increment per year.

The initial budget estimates were updated three times, providing 4-stage budget recourse decisions.

Considerations of Project Implementation Options

Segment-based project implementation option: selecting projects by roadway segment

Corridor-based project implementation option: selecting projects in corridors I-64, I-65, I-69, I-70, I-74, I-80, I-89, and I-94.
Computational Study Results
- Comparison of Total Benefits and Matching Rates of Selected Projects

### Comparison of Total Benefits of Selected Projects

<table>
<thead>
<tr>
<th>Budget</th>
<th>Total Benefits (in 1990, Billion Dollars)</th>
<th>Project Benefits by Highway System Goal</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Project Implementation Option</td>
<td>Agency Cost</td>
<td>VOC</td>
</tr>
<tr>
<td>Deterministic</td>
<td>Segment-based</td>
<td>9.78</td>
<td>4.78</td>
</tr>
<tr>
<td></td>
<td>Corridor-based</td>
<td>9.34</td>
<td>4.3</td>
</tr>
<tr>
<td></td>
<td>Deferrment-based</td>
<td>9.19</td>
<td>4.99</td>
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<tr>
<td>Stochastic</td>
<td>Segment-based</td>
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<td>4.86</td>
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<td>Corridor-based</td>
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<tr>
<td></td>
<td>Deferrment-based</td>
<td>9.27</td>
<td>5.03</td>
</tr>
</tbody>
</table>

| Deterministic         | Average                                  | 9.87        | 4.72 | 3.27     | 15.29  | 4.17        | 37.3  |
|                       | Stochastic                               | 9.96        | 4.77 | 3.30     | 15.42  | 4.22        | 37.7  |
|                       | Average                                  | 10.27       | 4.64 | 3.36     | 15.71  | 4.11        | 38.1  |

### Comparison of Consistency Matching Rates of Selected Projects

<table>
<thead>
<tr>
<th>Budget</th>
<th>Comparison Method</th>
<th>Average Number of Projects Authorized by Indiana DOT Authorization</th>
<th>Match with Indiana DOT Authorization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Project Implementation Option</td>
<td>Number Selected</td>
<td>Percent</td>
</tr>
<tr>
<td>Deterministic</td>
<td>Segment-based</td>
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<td>5,050</td>
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<tr>
<td></td>
<td>Corridor-based</td>
<td>5,964</td>
<td>4,955</td>
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<tr>
<td></td>
<td>Deferrment-based</td>
<td>6,038</td>
<td>5,064</td>
</tr>
<tr>
<td>Stochastic</td>
<td>Segment-based</td>
<td>6,023</td>
<td>5,059</td>
</tr>
<tr>
<td></td>
<td>Corridor-based</td>
<td>6,015</td>
<td>5,004</td>
</tr>
<tr>
<td></td>
<td>Deferrment-based</td>
<td>6,024</td>
<td>5,051</td>
</tr>
<tr>
<td>Deterministic budget</td>
<td>Average</td>
<td>6,006</td>
<td>5,023</td>
</tr>
<tr>
<td>Stochastic budget</td>
<td>Average</td>
<td>6,021</td>
<td>5,038</td>
</tr>
<tr>
<td>Average</td>
<td>Segment-based</td>
<td>6,020</td>
<td>5,055</td>
</tr>
<tr>
<td></td>
<td>Corridor-based</td>
<td>5,990</td>
<td>4,980</td>
</tr>
<tr>
<td></td>
<td>Deferrment-based</td>
<td>6,031</td>
<td>5,058</td>
</tr>
</tbody>
</table>

| Projects Authorized by Indiana DOT | 6,341           |
| Projects Matched for All Project Section Strategies | 4,656 | 73.4% |
Needed Model Enhancements

The proposed stochastic model addressing budget constraints by program category and by year, project tie-ins, and budget uncertainty is discussed.

Model enhancements are needed for:

- Adding chance constraints for expected infrastructure conditions and system operations service levels after project implementation
- Incorporating constraints for maximum allowable risks in the benefits of interdependent projects that would facilitate tradeoff analysis across different types of assets. This will help answer the following critical questions:
  - What happens if there is an across the board “x” percent decrease in both pavement and bridge investment levels?
  - What happens if funding is increased for the bridge program by “y” percent and there is a corresponding reduction in the pavement program?
Addressing Risks of Project Benefits in Trade-off Analysis

- Difference between risk and uncertainty
  - Risk involves objective probabilities and measurable quantities
  - Uncertainty involves subjective probabilities and immeasurable quantities

- Financial analysts and engineers have long dealt with the problems of managing, mitigating, and minimizing risk. Among the techniques used are mean-variance analysis, Value at Risk (VaR) and Stochastic Dominance

- Selecting projects for transportation asset management is similar to selecting stocks for a portfolio. Instead of stocks, we have highway projects. We will primarily limit our
Augmenting the Stochastic Model into Two-Phase Optimization

- **Phase I Optimization**: Find Minimum of Risks of Project Benefits
  - Markowitz mean-variance model formulation
    \[
    \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \text{cov}(b_i, b_j)
    \]
    Min
    \[
    \sum_{i=1}^{n} x_i \leq B = 100\%, \ x_i \geq 0, \text{and } E(b_i) \geq B_i
    \]
  
  where \( x_i \) is the proportion of our budget in dollar that are invested in project \( i \), \( b_i \) is the benefits of project \( i \), \( B_i \) is the threshold benefits of project \( i \), \( B \) is budget constraint, and \( i = 1, 2, ..., n \).

- **Phase II Optimization**: Use Optimal Value of the Objective Function from Phase I as Upper Bound Constraint of Risks of Project Benefits added to the Proposed Stochastic Model.
Project Benefits, Costs, and Covariance in the Markowitz Model

- **Proportion of Budget to be Used by a Project**
  
<table>
<thead>
<tr>
<th>Project</th>
<th>Benefits</th>
<th>Costs</th>
<th>Proportion of Obtainable Budget</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$b_1$</td>
<td>$C_1$</td>
<td>$X_1 = C_1/B$</td>
</tr>
<tr>
<td>2</td>
<td>$b_2$</td>
<td>$C_2$</td>
<td>$X_2 = C_2/B$</td>
</tr>
<tr>
<td>3</td>
<td>$b_3$</td>
<td>$C_3$</td>
<td>$X_3 = C_3/B$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>$\sum_{i=1}^{N} X_i \leq B$</td>
</tr>
<tr>
<td>N</td>
<td>$b_N$</td>
<td></td>
<td>$X_N = C_N/B$</td>
</tr>
</tbody>
</table>

- **Covariance of Benefits for Each Pair of Projects**

  \[
  \text{COV}(b_i, b_j) = E(b_i b_j) - E(b_i) E(b_j) = \sum_{S=1}^{3} \sum_{T=1}^{3} b_{i,S} b_{j,T} P(b_{i,S}, b_{j,T}) - \left[ \sum_{S=1}^{3} b_{i,S} P(b_{i,S}) \right] \left[ \sum_{T=1}^{3} b_{j,T} P(b_{j,T}) \right]
  \]

<table>
<thead>
<tr>
<th>$P_i$</th>
<th>$b_{i,L}$</th>
<th>$b_{i,M}$</th>
<th>$b_{i,H}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_j$</td>
<td>$P(b_{i,L}, b_{j,L})$</td>
<td>$P(b_{i,M}, b_{j,L})$</td>
<td>$P(b_{i,H}, b_{j,L})$</td>
</tr>
<tr>
<td></td>
<td>$P(b_{i,L}, b_{j,M})$</td>
<td>$P(b_{i,M}, b_{j,M})$</td>
<td>$P(b_{i,H}, b_{j,M})$</td>
</tr>
<tr>
<td></td>
<td>$P(b_{i,L}, b_{j,H})$</td>
<td>$P(b_{i,M}, b_{j,H})$</td>
<td>$P(b_{i,H}, b_{j,H})$</td>
</tr>
<tr>
<td></td>
<td>$P(b_{i,L})$</td>
<td>$P(b_{i,M})$</td>
<td>$P(b_{i,H})$</td>
</tr>
</tbody>
</table>
Wolfe’s LP Formulation for Solving the Markowitz Model

- Markowitz mean-variance model can be re-written in its general form:
  \[ \text{Objective: } \min z(x) = -cx + (1/2)x^\top Qx \]
  \[ \text{Subject to } Ax \geq b, \ x \geq 0. \]

where \( c \) = coefficient vector of the decision vector \( x \), \( x = [x_1, x_2, \ldots, x_N]^\top \), \( Q \) = positive definite matrix for the coefficients of the quadratic terms, \( A \) = vector of expected benefits of \( N \) projects, \( A = [a_1, a_2, \ldots, a_N]^\top \), \( b \) = threshold benefits of \( N \) projects, \( b = [b_1, b_2, \ldots, b_N]^\top \).

- As all variables \( x_1, x_2, \ldots, x_N \) are nonnegative, the Wolfe’s method could be adopted for solving a LP formulation derived from the Markowitz mean-variance model as follows:
  \[ \text{Objective: } \min w = a_1 + a_2 + \ldots + a_k \]
  \[ \text{Subject to } Qx - e + A^\top y = c^\top \]
  \[ Ax - e' = b \]
  \[ x \geq 0. \]

where \( a_1, a_2, \ldots, a_k \) = Non-negative artificial variables, \( e, e' \) = Non-negative excessive variable, and \( y \) = dual variables.
The Wolfe’s Modified Simplex Algorithm

Step 1: Modify the constraints so that the right-hand side of each constraint is non-negative. This requires that each constraint with a negative right-hand side be multiplied through by -1.

Step 2: Identify each constraint that is now an “=” or “≥” constraint.

Step 3: Cover each inequality constraint to the standard form. If constraint \( i \) is a “≤” constraint, add a slack variable \( s_i \). If constraint \( i \) is a “≥” constraint, add an excessive variable \( e_i \).

Step 4: For each “=” or “≥” constraint identified in Step 2, add an artificial variable \( a_k \).

Step 5: Solve for the LP by satisfying the complementary slackness requirements: \( ye' = 0 \) and \( ex = 0 \).

If the optimal value \( w > 0 \), the LP has no feasible solution. The solution \( x \) to which \( w = 0 \) is the optimal solution to the original Markowitz mean-variance model.
Concluding Remarks

- An improved stochastic model, along with an efficient heuristic algorithm, is introduced to address budget uncertainty and project implementation option issues in systemwide highway project selection.

- Computational study reveals that the stochastic model is able to determine the best project implementation option aimed to achieve the highest overall return on investments.

- The stochastic model needs to be further enhanced as two-phase optimization by addressing risks of project benefits to rigorously carry out cross-asset trade-off analysis.

- The Markowitz mean-variance model could be employed to find the upper bound of the overall allowable risks to be used as additional constraints to augment the stochastic model.
Bio-Sketch of Zongzhi Li

**Education**
- Chang’an University (B.E.)
- Purdue University
  - M.S.C.E. and Ph.D. in Transportation (Advisor: Kumares C. Sinha, U.S. NAE Member)
  - M.S.I.E. in Optimization (Advisor: Thomas L. Morin)

**Professional Experience**
- Two World Bank Financed Highway Projects
- From 2006, nine major research projects funded by ASCE, FHWA, Illinois DOT, Indiana DOT, U of Wisc MRUTC/CFIRE, Purdue JTRP, and Galvin Congestion Initiative (over $1.8 million grants)

**Research Interests**
- Transportation systems analysis, evaluation, and asset management
- Statistical and econometric methods for transportation infrastructure performance modeling and safety analysis
- Optimization, and risk and uncertainty modeling for transportation infrastructure systems and dynamic traffic networks.