Selling to Strategic Customers in the Presence of Consumption Network Externalities

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My background

- Assistant Professor of Management Science, Stuart School, since Fall 2007

- Ph.D. in Operations Management

- Research interests:
  - Operations and marketing issues, considering strategic consumer behavior, social network effects, etc.
  - Technological innovation management

- Research methodologies:
  - Optimization (deterministic, dynamic and stochastic)
  - Game Theory
  - Economic Models
Outline

- Background and motivation

- Consumers have full rationality
  - Equilibrium
  - Value of quantity commitment
  - Vertically decentralized channel
  - Horizontally decentralized channel

- Consumers have bounded rationality

- Concluding remarks
Background: Newsvendor problem

- Demand is uncertain, and the seller has to order/make a certain amount of quantity before demand is realized.
- Sequence of events:

  - Seller places an order (or makes) $Q$ at cost $c$.
  - Demand is known and sales occur at price $p$.
  - If any left over inventory, salvage at price $v$.

**Variable**
- Production/Ordering Cost $c$ ($35$)

**Selling Price** $p$ ($125$)

**Salvage Value** $v$ ($20$)
Background: Newsvendor problem

- Basic trade-off: stocking too much vs. stock too little

- The optimal $Q^*$ is achieved when expected overstocking cost = expected understocking cost

- Demand $X$ follows cdf $F$

\[ C_o = c - v: \text{overstocking cost} \]
\[ F(Q): \text{service level} \]
\[ C_u = p - c: \text{understocking cost} \]

\[ C_o \cdot F(Q) = C_u \cdot [1 - F(Q)] \]

\[ F(Q^*) = \frac{C_u}{C_u + C_o} \]
A decentralized channel, i.e., the retailer orders from a supplier at a wholesale price \( w \).

It has been shown that in a decentralized channel (under wholesale price contracts), the optimal order quantity is lower than the system-optimal quantity: \( Q' < Q^* \). That is, the decentralized channel performs worse (i.e., channel is not coordinated).

This is due to "Double marginalization" effects.
Background: Newsvendor problem

- How to motivate the retailer to order the system-optimal quantity Q*?

- Solutions: Instead of whole-sale price contracts, buy-back, revenue-sharing, etc., can achieve channel coordination. (The idea is to have the supplier share some risk of the channel.)

- **BUT!**
  A basic assumption of the newsvendor model is that customers are not strategic!
  - They don’t look forward, i.e., wait for price markdown
  - Retail price $p$ (consumers’ willingness-to-pay) is fixed, and independent of the sales quantity.
Motivation

- For many products, a consumer’s willingness-to-pay depends upon the total number of other consumers – *Consumption Network Externalities*.

- Their willingness-to-pay (retailer price $p$) can increase or decrease with the total expected sales, $\min(X, Q)$
  
  - *Positive* externalities: computer games, road navigation systems, movie DVDs
  
  - *Negative* externalities: fashion products

- For the seller:
  - How much $Q$ to order because now the price it can charge depends upon the expected sales quantity, i.e., $\min(Q, X)$?
  - Still, centralized channels overperform decentralized ones?
Model

- Full rationality assumptions: Rational Expectations (RE) theory: Economic outcomes do not differ systematically from what people expect them to be.

- In our problem, RE means in equilibrium, consumers’ willingness-to-pay equals, i.e.,

\[ p = v + \gamma \cdot E[\min(X, Q)] \]

where \( v \) is the intrinsic value of the product, \( \gamma \in (-\infty, +\infty) \) denotes the strength of network externalities.

\( p \) expected sale quantity
Centralized Channel

- The seller determines the order quantity by solving:

\[
\text{Maximize } p \mathbb{E}[\min(X, Q)] - cQ
\]

The retail price is given by

\[
p = E[v + \gamma \cdot \min(X, Q)]
\]

- This leads to

**Proposition 1** (1) There is a unique RE equilibrium \((p_c, Q_c)\), where the quantity \(Q_c\) is the larger solution (or unique solution) of

\[
[v + \gamma S(Q)] \overline{F}(Q) - c = 0
\]

and the price is: \(p_c = v + \gamma S(Q_c)\); and

(2) \(dQ_c/dc < 0\).
Centralized Channel with quantity commitment

- Q: Can the seller do better if the seller announces a fixed selling quantity Q and is able to commit to this quantity.

The seller’s problem now becomes

$$\max_Q E[v + \gamma \cdot \min(X,Q)] \cdot E[\min(X,Q)] - cQ$$

- This leads to

**Proposition 2**  If the seller can make credible quantity commitment,

(1) The seller’s optimal selling quantity, $Q_o$, is the larger solution (or unique solution) of

$$[v + 2\gamma S(Q)] \cdot F(Q) - c = 0.$$  

and the optimal price is $p_o = v + \gamma S(Q)$; and

(2) When $\gamma < 0$, $Q_o < Q_c$, and $\Pi_o > \Pi_c$; when $\gamma > 0$, $Q_o > Q_c$, and $\Pi_o > \Pi_c$.

With quantity commitment, the seller achieves a higher profit!
Vertically Decentralized Channel

- One supplier, one retailer
- wholesale-price contracts
- Wholesale price contract: Retailer orders from supplier at the wholesale price $w$.

Retailer maximizes: \[ \prod_w^r = p \cdot E[\min(X, Q)] - wQ \]

Supplier maximizes: \[ \prod_w^m = Q \cdot (w - c) \]

- Q: Can they achieve the system optimum, or even the optimum with quantity commitment?

**Proposition 3**

1. When $\gamma < 0$, there exists some $w^* \in (c, v)$ such that when $w = w^*$, $\Pi_w (w^*) = \Pi_q^*$, and for $w \in (c, w^*)$, $\Pi_w (w) > \Pi_c$.

2. When $\gamma > 0$, for any $w \in (c, v)$, $\Pi_w (w) < \Pi_c < \Pi_q^*$.

A vertically decentralized channel over-performs the centralized one without quantity commitment when the externality effect is negative.

Double marginalization effect plays a positive role!
Vertically Decentralized Channel

- Instead of wholesale price contracts, can other contracts achieve coordination?

**Proposition 4** When $\gamma < 0$, under the buy-back contract,

$$w_b = \left(1 - \frac{\lambda}{\lambda^*}\right)p^* + \frac{\lambda}{\lambda^*}w^*, \quad b = \left(1 - \frac{\lambda}{\lambda^*}\right)p^*,$$

the optimal profit $\Pi_q$ under quantity commitment is achieved and the retailer’s share is $\lambda \Pi_q$, where $\lambda \in [0, 1]$. When $\gamma > 0$, buy-back contracts cannot achieve quantity commitment profit $\Pi_q$.

**Proposition 5** For any $\gamma < 0$ or $\gamma > 0$, under the revenue-sharing contract,

$$\eta = 1 - \frac{\lambda}{1 + \frac{c\gamma S(Q_q)}{\gamma S(Q_q) + p_q} \frac{Q_q}{\Pi_q}}, \quad w_s = c \frac{\lambda}{1 + \frac{c\gamma S(Q_q)}{\gamma S(Q_q) + p_q} \frac{Q_q}{\Pi_q}} \frac{v + \gamma S(Q_q)}{v + 2\gamma S(Q_q)},$$

the optimal profit $\Pi_q$ under quantity commitment is achieved and the retailer’s share is $\lambda \Pi_q$, where $\lambda \in (0, 1]$. 

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Horizontally Decentralized Channel

- Instead of a single retailer, what is there are multiple competing retailers?
- Model setting: Two retailers each face a random demand \( X_i \sim F_i(.) \), and each places an order \( Q_i \).

Consumers’ reservation price is now given by:

\[
E[v + \gamma \cdot \min(X_1 + X_2, Q_1 + Q_2)]
\]

**Proposition 6** There exists a positive critical \( \hat{\gamma} > 0 \), such that for \( \gamma \leq \hat{\gamma} \):

(1) In a decentralized system, there is a unique RE equilibrium \((p_h^d, Q_1^d, Q_2^d)\), where the quantities \((Q_1^d, Q_2^d)\) solve

\[
\begin{align*}
\{v + \gamma E[(X_1 + X_2) \land (Q_1 + Q_2)]\} 
& \left[1 - \int_0^{Q_1} F_2 (Q_1 + Q_2 - x_1) f_1 (x_1) \, dx_1\right] = c \\
\{v + \gamma E[(X_1 + X_2) \land (Q_1 + Q_2)]\} 
& \left[1 - \int_0^{Q_2} F_1 (Q_1 + Q_2 - x_2) f_2 (x_2) \, dx_2\right] = c
\end{align*}
\]

and the price is: \( p_h^d = v + \gamma E[(X_1 + X_2) \land (Q_1^d + Q_2^d)] \);

(2) In the decentralized system, the industry inventory level is higher than that in the centralized one, i.e.,

\[ Q_1^d + Q_2^d > Q_1^c + Q_2^c, \]

and \( p_h^d < p_h^c \).
Bounded Rationality and Consumer Learning

- Consumers do not necessarily have a full rationality, and they learn through repeated experiences.

- In each period $t$, consumers’ reservation price is given by
  \[ p_t = E[v + \gamma \hat{\xi}_t] \]
  and at the end of period $t$, consumers update its belief on the sales quantity
  \[ \hat{\xi}_{t+1} = \alpha E[\min(Q_t, X_t)] + (1 - \alpha)\hat{\xi}_t \]

- The seller now faces a dynamic optimization problem, with the Bellman equation:
  \[ V(\hat{\xi}) = \max_{Q} \prod Q (Q, \hat{\xi}) + \delta \cdot V(\alpha E[\min(Q_t, X_t)] + (1 - \alpha)\hat{\xi}) \]
Bounded Rationality and Consumer Learning

Quantities $Q_t^*$ and beliefs $\hat{\xi}_t$ are governed by the following dynamic process:

$$Q_1^* = Q^*(\hat{\xi}_1), \quad \hat{\xi}_2 = \alpha S(Q_1^*) + (1 - \alpha)\hat{\xi}_1,$$
$$Q_2^* = Q^*(\hat{\xi}_2), \quad \hat{\xi}_3 = \alpha S(Q_2^*) + (1 - \alpha)\hat{\xi}_2, \ldots$$
$$Q_t^* = Q^*(\hat{\xi}_t), \quad \hat{\xi}_{t+1} = \alpha S(Q_t^*) + (1 - \alpha)\hat{\xi}_t, \ldots$$

We are interested in if the optimal quantities and beliefs converge to a long run steady state?
Bounded Rationality and Consumer Learning

Proposition 7 When \( \gamma > \hat{\gamma} \), there exists some threshold \( \hat{\xi} \) such that \( Q^*_i \rightarrow \hat{Q} \) and \( \hat{\xi}_i \rightarrow \hat{\xi} \), where \( \hat{\gamma} < 0, \hat{\xi} = S(\hat{Q}) \) and \( Q > 0 \) is the larger solution (or unique solution) of

\[
\left[ v + \left(1 + \frac{\alpha \delta}{1 - (1 - \alpha) \delta}\right) \gamma S(Q) \right] \bar{F}(Q) - c = 0. \tag{11}
\]

Proposition 8 Let \( Q^*[\gamma] \) be the RE equilibrium quantity in the base model when the externality coefficient is \( \gamma \). Then the equilibrium quantity under consumer learning is given by

\[
\hat{Q} = Q^* \left[ \left(1 + \frac{\alpha \delta}{1 - (1 - \alpha) \delta}\right) \gamma \right]. \tag{12}
\]

- In this adaptive learning model, there is a unique long-run equilibrium as long as the network externality is positive or not too negative.

- The long-run equilibrium in the dynamic model approaches the commitment benchmark when the discount factor \( \delta \) approaches 1 from below, i.e., when the seller’s reputational concern is strong enough.
Concluding Remarks

- **Value of quantity commitment**
  - With positive (negative) externality effect, the seller’s profit can be enhanced if he is able to commit to stock a higher (lower) quantity than that in the RE equilibrium.

- **How does strategic consumer behavior affect supply chain contracting decisions**
  - Under the presence of negative externalities, a vertically decentralized channel with a wholesale-price contract may perform strictly better than a centralized one.

  - Via properly structured contracts, a decentralized channel may achieve the quantity commitment outcomes of the centralized channel.
    - Under negative externalities, we expect that buy-back contracts can be used to achieve the quantity commitment outcomes.
    - Under positive externalities, revenue-sharing contracts may achieve the desirable quantity commitment outcomes.

  - Under the presence of positive externalities, a horizontally decentralized channel may perform strictly better than a centralized one.
Concluding Remarks

- Seller’s optimal strategy when rationality is bounded and consumers learn
  - When consumers learn through repeated experiences, the seller’s reputational concern may serve as a surrogate for commitment power.
Q&A