Selling to Strategic Customers in the Presence of Consumption Network Externalities

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My background

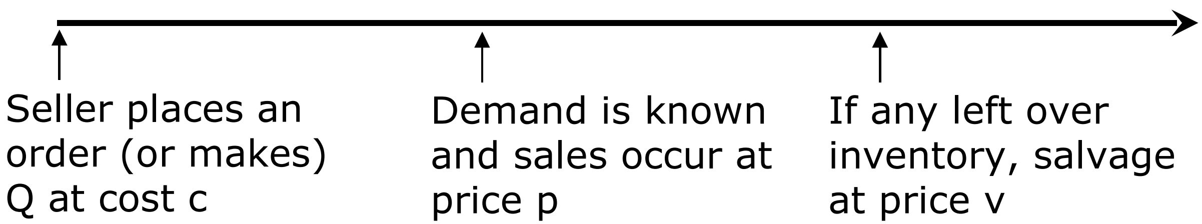
- Assistant Professor of Management Science, Stuart School, since Fall 2007
- Ph.D. in Operations Management
- Research interests:
  - Operations and marketing issues, considering strategic consumer behavior, social network effects, etc.
  - Technological innovation management
- Research methodologies:
  - Optimization (deterministic, dynamic and stochastic)
  - Game Theory
  - Economic Models

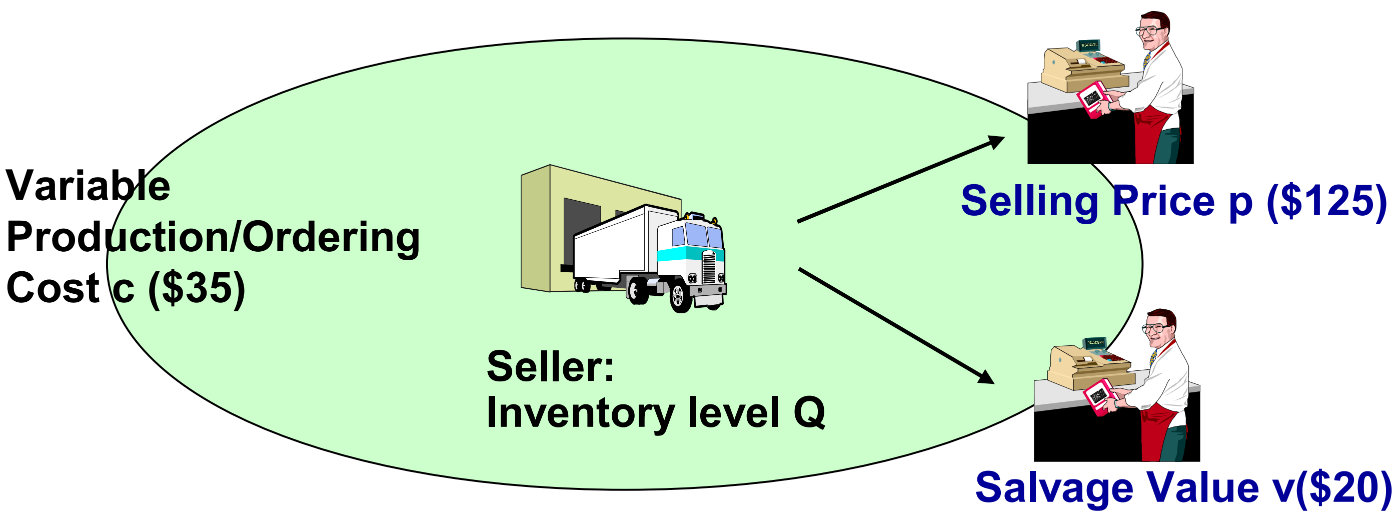
Outline

- Background and motivation
- Consumers have full rationality
  - Equilibrium
  - Value of quantity commitment
  - Vertically decentralized channel
  - Horizontally decentralized channel
- Consumers have bounded rationality
- Concluding remarks

Background: Newsvendor problem

- Demand is uncertain, and the seller has to order/make a certain amount of quantity before demand is realized
- Sequence of events:



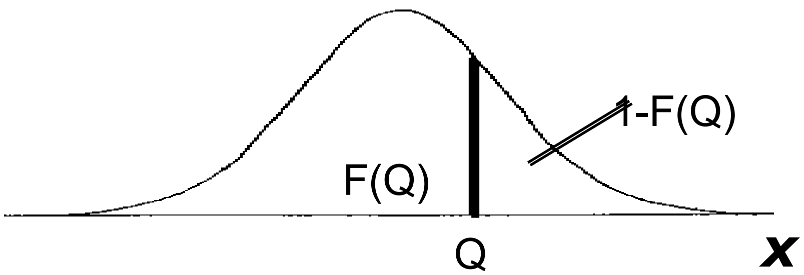


Background: Newsvendor problem

- Basic trade-off:
  stocking too much vs. stock too little

- The optimal Q* is achieved when
  expected overstocking cost = expected understocking cost

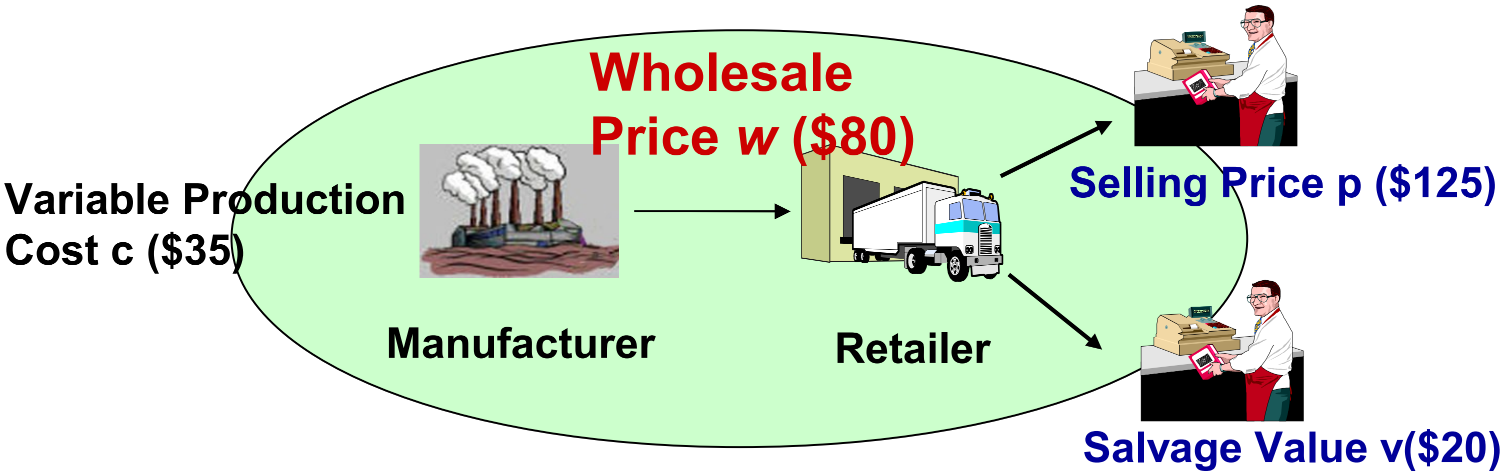
- Demand *X* follows cdf *F*



- C_O=c-v: overstocking cost C_u=p-c: understocking cost
  F(Q): service level
  $C_O * F(Q) = C_u * [1- F(Q)]$ ⇒ $F(Q^*) = \frac{C_u}{C_u + C_o}$

Background: Newsvendor problem

- A decentralized channel, i.e., the retailer orders from a supplier at a wholesale price w



- It has been shown that in a decentralized channel (under wholesale price contracts), the optimal order quantity is lower than the system-optimal quantity: Q' < Q*.

  That is, the decentralized channel performs *worse* (i.e., channel is *not coordinated*).
- This is due to "*Double marginalization*" effects.

Background: Newsvendor problem

- How to motivate the retailer to order the system-optimal quantity Q*?

- Solutions: Instead of whole-sale price contracts, buy-back, revenue-sharing, etc., can achieve channel coordination. (The idea is to have the supplier share some risk of the channel.)

- **BUT**!
  A basic assumption of the newsvendor model is that *customers are not strategic*!
  - They don't look forward, i.e., wait for price markdown
  - **Retail price *p* (consumers' willingness-to-pay) is fixed, and independent of the sales quantity.**

Motivation

- For many products, a consumer’s willingness-to-pay depends upon the total number of other consumers – ***Consumption Network Externalities***.

- Their willingness-to-pay (retailer price p) can increase or decrease with the total expected sales, min(X,Q)

  - *Positive* externalities: computer games, road navigation systems, movie DVDs

  - *Negative* externalities: fashion products

- For the seller:
  - How much Q to order because now the price it can charge depends upon the expected sales quantity, i.e., min(Q, X)?
  - Still, centralized channels overperform decentralized ones?

Model

- Full rationality assumptions:
  Rational Expectations (RE) theory: Economic outcomes do not differ systematically from what people expect them to be.

- In our problem, RE means in equilibrium, consumers' willingness-to-pay equals, i.e.,

$$p = v + \gamma \cdot \underbrace{E[\min(X, Q)]}_{\text{expected sale quantity}}$$

where *v* is the intrinsic value of the product, $\gamma \in (-\infty, +\infty)$ denotes the strength of network externalities.

Centralized Channel

- The seller determines the order quantity by solving:

$$\underset{Q}{Maximize} \quad pE[\min(X,Q)] - cQ$$

  The retail price is given by

$$p = E[v + \gamma \cdot \min(X,Q)]$$

- This leads to

Proposition 1 *(1) There is a unique RE equilibrium (p_c, Q_c), where the quantity Q_c is the larger solution (or unique solution) of*

$$[v + \gamma S(Q)]\,\overline{F}(Q) - c = 0 \qquad (1)$$

and the price is: $p_c = v + \gamma S(Q_c)$; and
(2) $dQ_c/dc < 0$.

Centralized Channel with quantity commitment

- Q: Can the seller do better if the seller announces a fixed selling quantity Q and is able to commit to this quantity.

  The seller's problem now becomes

$$\max_{Q} \quad E[v + \gamma \cdot \min(X,Q)] \cdot E[\min(X,Q)] - cQ$$

- This leads to

Proposition 2 *If the seller can make credible quantity commitment,*
(1) The seller's optimal selling quantity, Q_q, is the larger solution (or unique solution) of

$$[v + 2\gamma S(Q)]\, \overline{F}(Q) - c = 0. \tag{2}$$

and the optimal price is $p_q = v + \gamma S(Q_q)$; and
(2) When $\gamma < 0$, $Q_q < Q_c$, and $\Pi_q > \Pi_c$; when $\gamma > 0$, $Q_q > Q_c$, and $\Pi_q > \Pi_c$.

⇨ With quantity commitment, the seller achieves a higher profit!

Vertically Decentralized Channel

- One supplier, one retailer
- wholesale-price contracts
- Wholesale price contract: Retailer orders from supplier at the wholesale price w.

  Retailer maximizes: $\Pi_w^r = p \cdot E[\min(X, Q)] - wQ$

  Supplier maximizes: $\Pi_w^m = Q \cdot (w - c)$

- Q: Can they achieve the system optimum, or even the optimum with quantity commitment?

Proposition 3 *(1) When $\gamma < 0$, there exists some $w^* \in (c, v)$ such that when $w = w^*$, $\Pi_w(w^*) = \Pi_q^*$, and for $w \in (c, w^*]$, $\Pi_w(w) > \Pi_c$.*
(2) When $\gamma > 0$, for any $w \in (c, v)$, $\Pi_w(w) < \Pi_c < \Pi_q^*$.

⇨ A vertically decentralized channel over-performs the centralized one without quantity commitment when the externality effect is negative.
Double marginalization effect plays a positive role!

Vertically Decentralized Channel

- Instead of wholesale price contracts, can other contracts achieve coordination?

Proposition 4 *When $\gamma < 0$, under the buy-back contract,*

$$w_b = \left(1 - \frac{\lambda}{\lambda^*}\right) p^* + \frac{\lambda}{\lambda^*} w^*, \; b = \left(1 - \frac{\lambda}{\lambda^*}\right) p^*,$$

the optimal profit Π_q under quantity commitment is achieved and the retailer's share is $\lambda\Pi_q$, where $\lambda \in [0,1]$. When $\gamma > 0$, buy-back contracts cannot achieve quantity commitment profit Π_q.

Proposition 5 *For any $\gamma < 0$ or $\gamma > 0$, under the revenue-sharing contract,*

$$\eta = 1 - \frac{\lambda}{1 + \frac{c\gamma S(Q_q)}{\gamma S(Q_q) + p_q} \frac{Q_q}{\Pi_q}}, \; w_s = c \frac{\lambda}{1 + \frac{c\gamma S(Q_q)}{\gamma S(Q_q) + p_q} \frac{Q_q}{\Pi_q}} \frac{v + \gamma S(Q_q)}{v + 2\gamma S(Q_q)},$$

the optimal profit Π_q under quantity commitment is achieved and the retailer's share is $\lambda\Pi_q$, where $\lambda \in (0,1]$.

Horizontally Decentralized Channel

- Instead of a single retailer, what is there are multiple competing retailers?
- Model setting: Two retailers each face a random demand $X_i \sim F_i(.)$, and each places an order Q_i. Consumers' reservation price is now given by:

$$E[v + \gamma \cdot \min(X_1 + X_2, Q_1 + Q_2)]$$

Proposition 6 *There exists a positive critical $\widehat{\gamma} > 0$, such that for $\gamma \leq \widehat{\gamma}$:*
(1) In a decentralized system, there is a unique RE equilibrium (p_h^d, Q_1^d, Q_2^d), where the quantities (Q_1^d, Q_2^d) solve

$$\begin{cases} \{v + \gamma E[(X_1 + X_2) \wedge (Q_1 + Q_2)]\}\left[1 - \int_0^{Q_1} F_2(Q_1 + Q_2 - x_1) f_1(x_1)\, dx_1\right] = c \\ \{v + \gamma E[(X_1 + X_2) \wedge (Q_1 + Q_2)]\}\left[1 - \int_0^{Q_2} F_1(Q_1 + Q_2 - x_2) f_2(x_2)\, dx_2\right] = c \end{cases} \quad (3)$$

and the price is: $p_h^d = v + \gamma E\left[(X_1 + X_2) \wedge (Q_1^d + Q_2^d)\right]$;
(2) In the decentralized system, the industry inventory level is higher than that in the centralized one, i.e.,

$$Q_1^d + Q_2^d > Q_1^c + Q_2^c,$$

and $p_h^d < p_h^c$.

Bounded Rationality and Consumer Learning

- Consumers do not necessarily have a full rationality, and they learn through repeated experiences.
- In each period t, consumers' reservation price is given by

$$p_t = E[v + \gamma \hat{\xi}_t]$$

and at the end of period t, consumers update its belief on the sales quantity

$$\hat{\xi}_{t+1} = \alpha E[\min(Q_t, X_t)] + (1 - \alpha)\hat{\xi}_t$$

- The seller now faces a dynamic optimization problem, with the Bellman equation:

$$V(\hat{\xi}) = \max_Q [\prod (Q, \hat{\xi}) + \delta \cdot V(\alpha E[\min(Q_t, X_t)] + (1 - \alpha)\hat{\xi})]$$

Bounded Rationality and Consumer Learning

- Quantities Q_t^* and beliefs $\hat{\xi}_t$ are governed by the following dynamic process:

$$Q_1^* = Q^*(\hat{\xi}_1), \quad \hat{\xi}_2 = \alpha S(Q_1^*) + (1-\alpha)\hat{\xi}_1,$$
$$Q_2^* = Q^*(\hat{\xi}_2), \quad \hat{\xi}_3 = \alpha S(Q_2^*) + (1-\alpha)\hat{\xi}_2, \ldots$$
$$Q_t^* = Q^*(\hat{\xi}_t), \quad \hat{\xi}_{t+1} = \alpha S(Q_t^*) + (1-\alpha)\hat{\xi}_t, \ldots$$

- We are interested in if the optimal quantities and beliefs converge to a long run steady state?

Bounded Rationality and Consumer Learning

Proposition 7 *When $\gamma > \widehat{\gamma}$, there exists some threshold $\dot{\xi}$ such that $Q_t^* \to \dot{Q}$ and $\hat{\xi}_t \to \dot{\xi}$, where $\widehat{\gamma} < 0$, $\dot{\xi} = S(\dot{Q})$ and $\dot{Q} > 0$ is the larger solution (or unique solution) of*

$$\left[v + \left(1 + \frac{\alpha\delta}{1-(1-\alpha)\delta}\right)\gamma S(Q)\right]\overline{F}(Q) - c = 0. \quad (11)$$

Proposition 8 *Let $Q^*[\gamma]$ be the RE equilibrium quantity in the base model when the externality coefficient is γ. Then the equilibrium quantity under consumer learning is given by*

$$\dot{Q} = Q^*\left[\left(1 + \frac{\alpha\delta}{1-(1-\alpha)\delta}\right)\gamma\right]. \quad (12)$$

- In this adaptive learning model, there is a unique long-run equilibrium as long as the network externality is positive or not too negative.
- The long-run equilibrium in the dynamic model approaches the commitment benchmark when the discount factor δ approaches 1 from below, i.e., when the seller’s reputational concern is strong enough

Concluding Remarks

- *Value of quantity commitment*
  - **With positive (negative) externality effect, the seller's profit can be enhanced if he is able to commit to stock a higher (lower) quantity than that in the RE equilibrium.**

- *How does strategic consumer behavior affect supply chain contracting decisions*
  - **Under the presence of negative externalities, a vertically decentralized channel with a wholesale-price contract may perform strictly better than a centralized one.**

  - Via properly structured contracts, a decentralized channel may achieve the quantity commitment outcomes of the centralized channel.
    - Under negative externalities, we expect that buy-back contracts can be used to achieve the quantity commitment outcomes.
    - Under positive externalities, revenue-sharing contracts may achieve the desirable quantity commitment outcomes.

  - **Under the presence of positive externalities, a horizontally decentralized channel may perform strictly better than a centralized one.**

Concluding Remarks

- *Seller’s optimal strategy when rationality is bounded and consumers learn*
  - **When consumers learn through repeated experiences, the seller’s reputational concern may serve as a surrogate for commitment power.**

Q&A