NONAUTONOMOUS DISCRETE DYNAMICS:
DISCRETIZATION AND TAYLOR APPROXIMATION

CHRISTIAN PÖTZSCHE
SCHOOL OF MATHEMATICS
UNIVERSITY OF MINNESOTA
MINNEAPOLIS, MN 55455

EMAIL: POETZSCH@umn.edu
HTTP://WWW.MATH.umn.edu/~POETZSCH/

Abstract. In this talk, we consider a class of nonautonomous
difference equations (recursions) possessing an attractive invariant
manifold in the extended state space. It is our aim to study the
behavior of this manifold, as well as of its corresponding asymptotic
phase under perturbation. Our framework is sufficiently general
to include discrete counterparts of (center-)unstable and inertial
manifolds.

Two main applications of such results for so-called invariant
fiber bundles will be discussed:

• They indicate that attractive invariant manifolds of evolu-
tionary equations persist under a large class of numerical
discretizations. Moreover, the rate of convergence can be
estimated. Similar results hold for the associated asymptotic
phase.

• We develop a method to calculate their Taylor approxima-
tion. Here, the desired Taylor coefficients are determined by
bounded solutions of a certain linear difference equation in
the space of multilinear mappings, instead of an algebraic
equation occurring in the classical autonomous theory. Even
in finite dimensions, such a technique is of crucial impor-
tance for an application of the reduction principle in a nonau-
tonous stability theory.