

# Sampling-Based Methods for Stochastic Optimization

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We consider stochastic optimization problems of the form

$$\min_{x \in X} \{g(x) := \mathbb{E}[G(x, \boldsymbol{\xi})]\}, \quad (1)$$

where  $X$  is a subset of  $\mathbb{R}^n$ ,  $\boldsymbol{\xi}$  is a random vector in  $\mathbb{R}^s$  and  $G : \mathbb{R}^n \times \mathbb{R}^s \mapsto \mathbb{R}$  is a real valued function. Oftentimes the expectation in (1) cannot be calculated exactly, particularly when  $G$  does not have a closed form. We consider a family  $\{\hat{g}_N(\cdot)\}$  of random approximations of the function  $g(\cdot)$ , each  $\hat{g}_N(\cdot)$  being defined as  $\hat{g}_N(x) := \frac{1}{N} \sum_{j=1}^N G(x, \boldsymbol{\xi}^j)$ , where  $\boldsymbol{\xi}^1, \dots, \boldsymbol{\xi}^N$  are samples from the distribution of  $\boldsymbol{\xi}$ . Given the family of estimators  $\{\hat{g}_N(\cdot)\}$ , one can construct the corresponding approximating program

$$\min_{x \in X} \hat{g}_N(x). \quad (2)$$

Let  $\hat{x}_N$  and  $\hat{\nu}_N$  denote respectively an optimal solution and the optimal value of (2). Then,  $\hat{x}_N$  and  $\hat{\nu}_N$  provide approximations respectively to an optimal solution  $x^*$  and the optimal value  $\nu^*$  of the true problem (1).

In this work we discuss the behavior of the approximating problem (2). In particular, we contrast results obtained under the assumption that the samples are independent and identically distributed with the situation in which they are not — which is motivated by the use of methods such as quasi-Monte Carlo and Latin hypercube sampling that aim at providing better pointwise estimates. We also discuss some algorithmic issues. We present numerical results illustrating the discussed ideas.