Numerical integration in high dimensions using digital nets

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Quasi-Monte Carlo rules have been used successfully for numerical integration. Such problems frequently occur in applications such as financial mathematics, statistics or physics.

In theoretical investigations one often considers integration over the high dimensional unit-cube, i.e.,

$$I_s(f) = \int_{[0,1]^s} f(\boldsymbol{x}) \mathrm{d}\boldsymbol{x},$$

which we approximate by

$$Q_{n,s}(f) = \frac{1}{n} \sum_{k=1}^{n} f(\boldsymbol{x}_k),$$

where $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n \in [0,1]^s$ are the quadrature points. For higher dimensions the question of how to make a sensible choice of quadrature points becomes increasingly difficult, with several important questions yet to be answered.

Two main construction methods for choosing those quadrature points have established themselves, one being called *lattice rules* and the other one goes by the name *digital nets*. The underlying theory between those two construction methods has been somewhat different and several useful results on (strong) tractability, which, in our case, is concerned with the behavior of quadrature rules in arbitrary high dimensions, have up to recently only been known for lattice rules.

In this talk we show results on (strong) tractability for digital nets which match the results obtained for lattice rules. The results are based on a far-reaching analogy of the underlying concepts, which in more recent results also allow us to cover further ground previously only reserved for the theory of lattice rules.