"Random geometric graph diameter in the unit ball"

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Abstract. Let $d \geq 2$ and n be positive integers, and $\lambda > 0$ a real number. Let V_n be a set of n points in the unit ball in $\mathbb{R}^d$ selected uniformly and independently at random. Define $G = G(\lambda, n)$ to be the graph with vertex set V_n, in which two vertices are adjacent provided their Euclidean distance is at most λ. G is an example of a *random geometric graph*, currently of great interest in modelling properties of wireless ad hoc networks and sensor networks. We compare the random geometric graph to its cousin, the Erdős-Rényi graph, in which each of $\binom{n}{2}$ possible edges is present independently with probability $p \in [0, 1]$.

The threshold for G to be connected is known to be $\lambda \sim (\frac{2(d-1)}{d} \frac{\ln n}{n})^{1/d}$. Many problems for networks, such as routing, broadcasting or throughput optimization, require connectivity. However, due to the difficulty of analyzing G in the supercritical region, often results are either not tight or leave a gap above the threshold. Using a blend of stochastic geometry and probabilistic combinatorics, we prove that the graph diameter of G is asymptotically $2/\lambda$ *as soon as G is connected.* We discuss implications for routing in wireless ad hoc networks.