Math 532 — Homework 9 — Due: Wednesday, April 8, 2015

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1. Let $\boldsymbol{u}$ and $\boldsymbol{v}$ be n-vectors and assume that $\boldsymbol{u}^T\boldsymbol{v} = 1$.
   (a) Show that $\mathsf{P} = \boldsymbol{u}\boldsymbol{v}^T$ is a projector.
   (b) Show that the projector P and its complementary projector $\mathsf{I} - \mathsf{P}$ have the same 2-norm, i.e.,
   $$\|\boldsymbol{u}\boldsymbol{v}^T\|_2 = \|\mathsf{I} - \boldsymbol{u}\boldsymbol{v}^T\|_2.$$
   Hint: This result can be established using the angle between subspaces.
   (c) Show that we actually do not need to compute these matrix norms. Instead, the vector norms suffice. Namely,
   $$\|\boldsymbol{u}\boldsymbol{v}^T\|_2 = \|\boldsymbol{u}\|_2\|\boldsymbol{v}\|_2.$$

2. Show that — as claimed on slide #142 — even if $\mathcal{M} \subseteq \mathcal{V}$ is not a subspace of the inner product space $\mathcal{V}$, its orthogonal complement $\mathcal{M}^\perp$ *is* a subspace of $\mathcal{V}$.

3. This is the numerical example for the URV factorization announced on slide #159.
   (a) Follow the general algorithm for the URV factorization given in the notes and apply it to the matrix
   $$\mathsf{A} = \begin{pmatrix} -4 & -2 & -4 & -2 \\ 2 & -2 & 2 & 1 \\ -4 & 1 & -4 & -2 \end{pmatrix}.$$
   (b) What are the orthonormal bases for the four fundamental subspaces for this specific A provided by its URV factorization?

4. Consider the matrix
   $$\mathsf{A} = \begin{pmatrix} -2 & 11 \\ -10 & 5 \end{pmatrix}.$$
   (a) Follow the general algorithm for the SVD given in the notes and apply it to A.
   (b) List the singular values of A, and specify the left and right singular vectors. Draw a picture of the unit ball in $\mathbb{R}^2$ and its image under A. Include the singular vectors and label them carefully.

5. (a) Show that for a square nonsingular matrix A we have $\mathsf{A}^\dagger = \mathsf{A}^{-1}$. This verifies the claim on slide #182.
   (b) Show that (for arbitrary A) $\left(\mathsf{A}^\dagger\right)^T = \left(\mathsf{A}^T\right)^\dagger$.