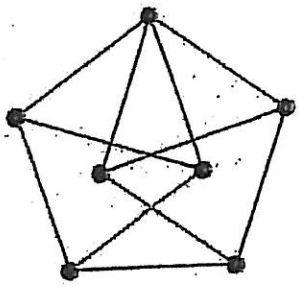
MATH 100 : Discrete Math Homework - Fall 2012

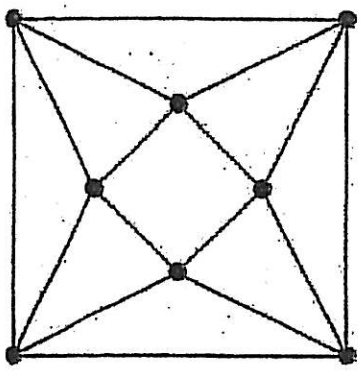
HW due by 4pm, Thursday, 11/8 in the mailbox of Prof. Hemanshu Kaul, Room 210, E1 bldg.
If you need clarification about a question, send him an email or stop by his office during his office hours. See his webpage at www.iit.edu/csl/am/faculty/kaul_hemanshu.shtml for contact information.

1. Refer to the last page of HW for this question. This page contains visual representations of two combinatorial identities of the form $A = B$. Both sides are counting area but from different perspectives. For both figures, do the following:
(i) carefully explain where each side of the identity is coming from;
(ii) carefully draw and explain how the figure should be extended to the next step, that is, from n to $n+1$.

2. Consider the graphs, K_4, K_5, K_6, $K_{2,3}$, $K_{3,3}$, $K_{4,3}$.
Use the applet at the website
mathdl.maa.org/images/upload_library/3/EnsleyPlanar/planarApplets.html
to explore the drawings of these 6 graphs (use the third option "3. Draw your own graph") and do the following questions.
(i) For each of these 6 graphs, either submit a planar drawing or a justification why the graph is not planar.
(ii) For each of these graphs that you claim are not planar, submit a drawing with the least number of crossings of edges.

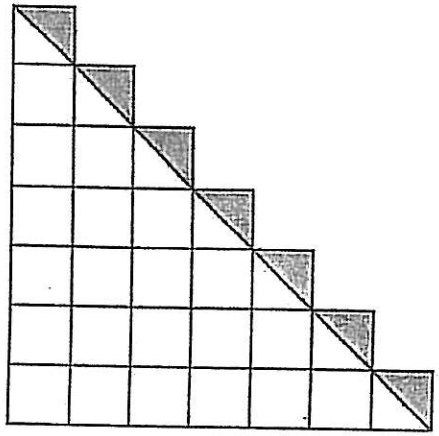
3. For each of the two graphs below, find a proper 4-coloring and explain why it can not be colored using 3 colors. (Hint: Try coloring it with 3 colors and get a contradiction by forcing the same color on two vertices with an edge between them.)





4. Construct an art gallery with n corners that can not be guarded by less than $\lfloor n/3 \rfloor$ guards. Your construction should work for any given $n = 3, 4, \ldots$ and you should explain why $\lfloor n/3 \rfloor$ guards are required for your art gallery.

Sums of Integers II

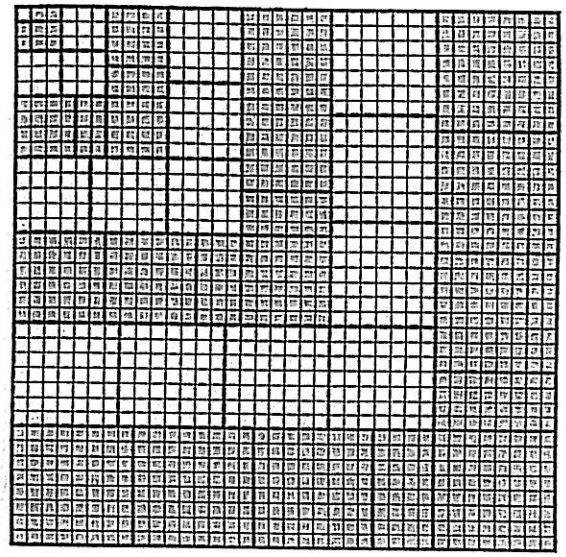


$$1+2+\cdots+n=\frac{n^2}{2}+\frac{n}{2}$$

—Ian Richards

Sums of Cubes II

$$1^3+2^3+3^3+\cdots+n^3=(1+2+3+\cdots+n)^2$$



—J. Barry Love