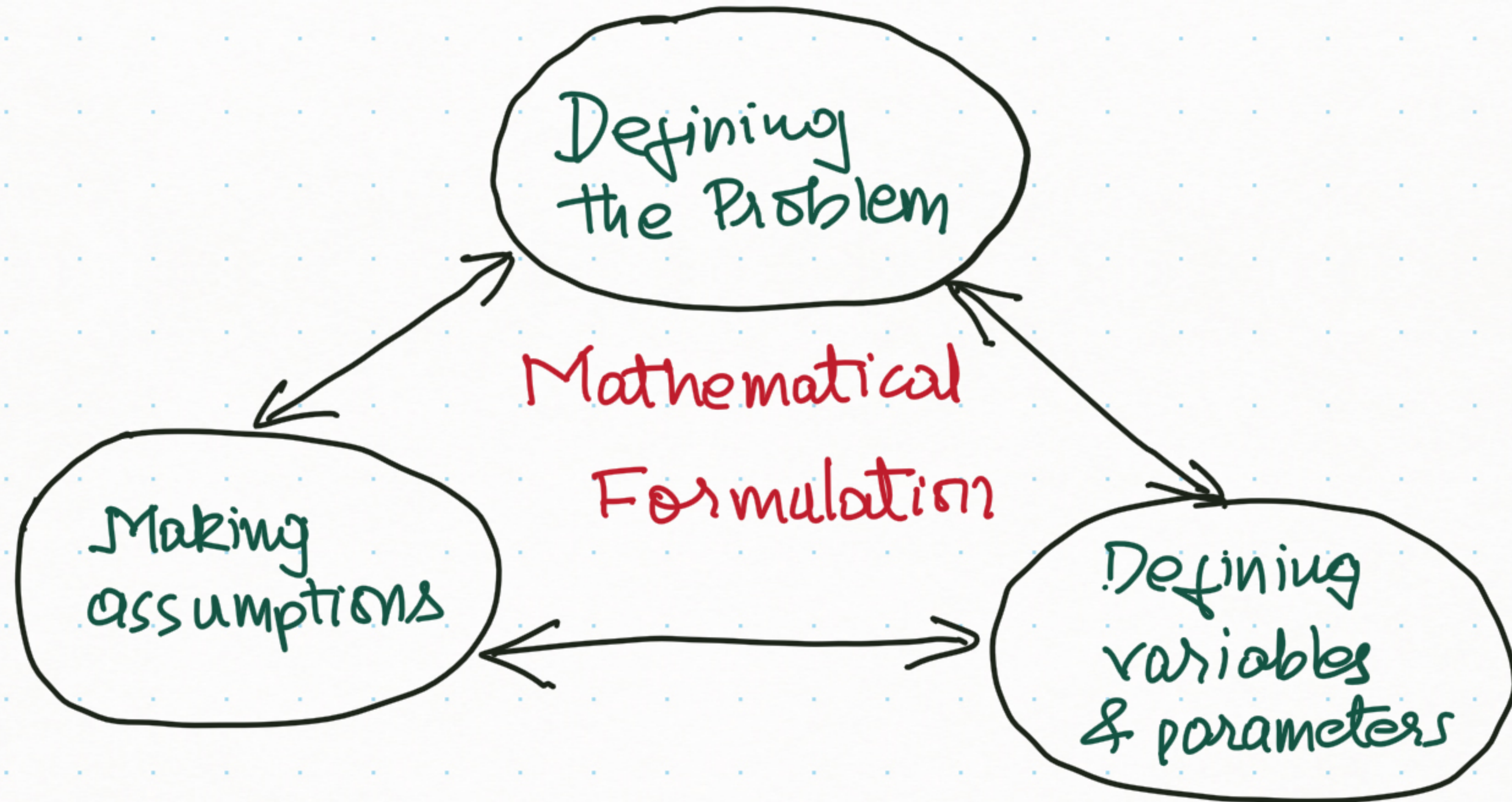


MATH 380

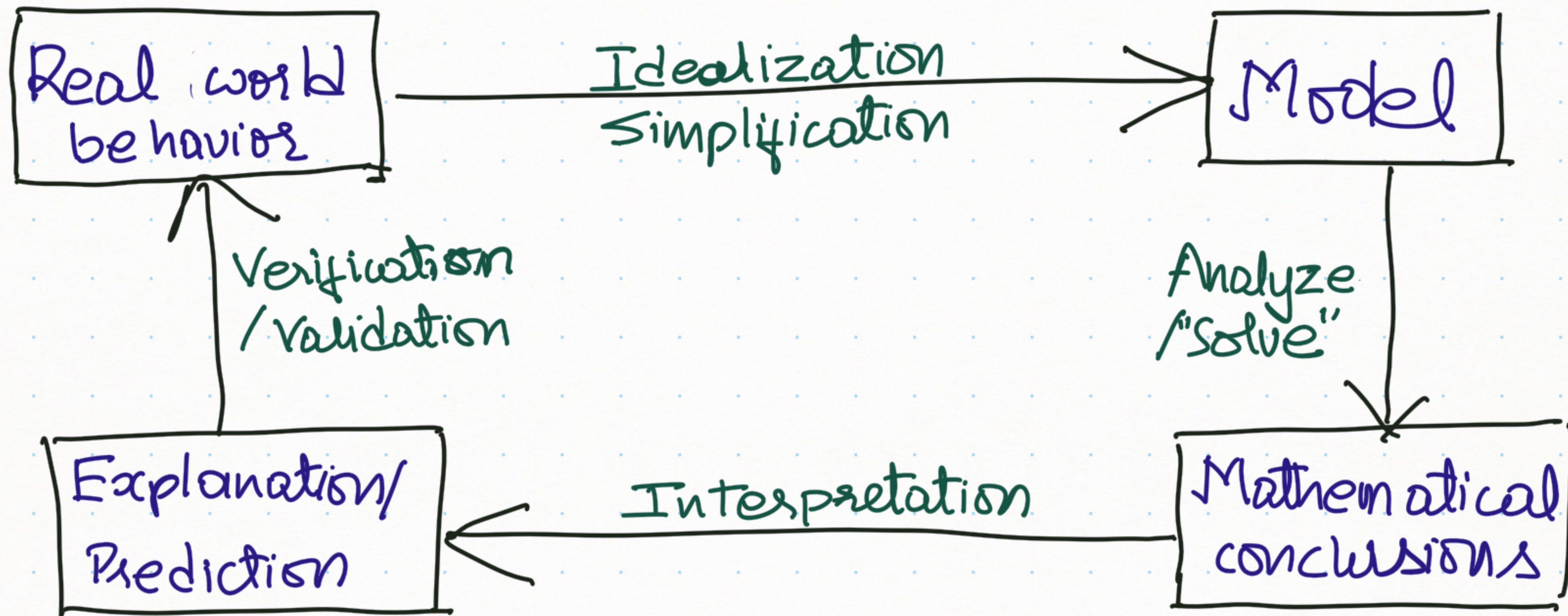
Hemanshu Kaul

kaul@iit.edu

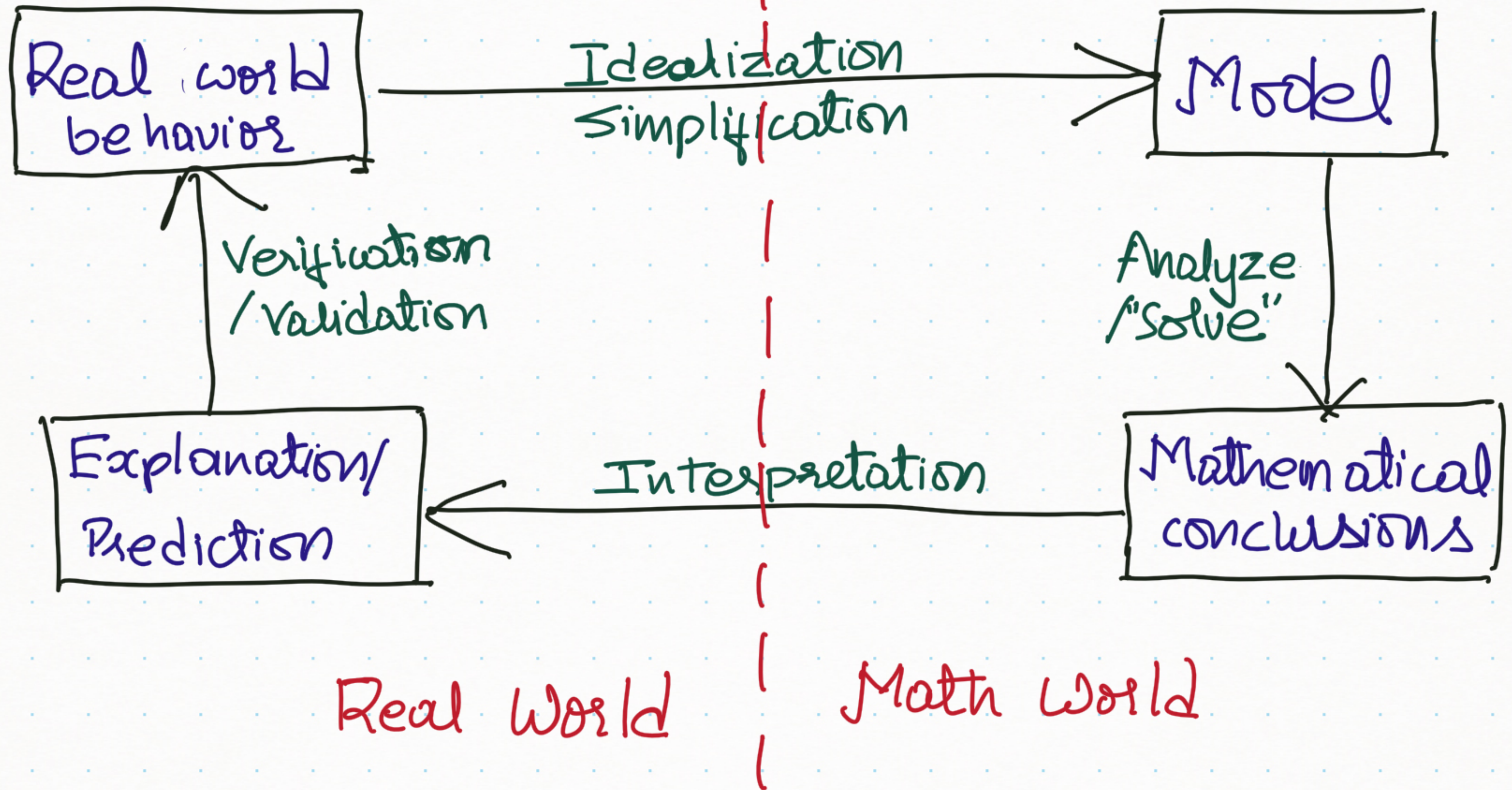
Mathematical Modeling



Mathematical Modeling



Mathematical Modeling



"Every model is wrong, but some are useful"

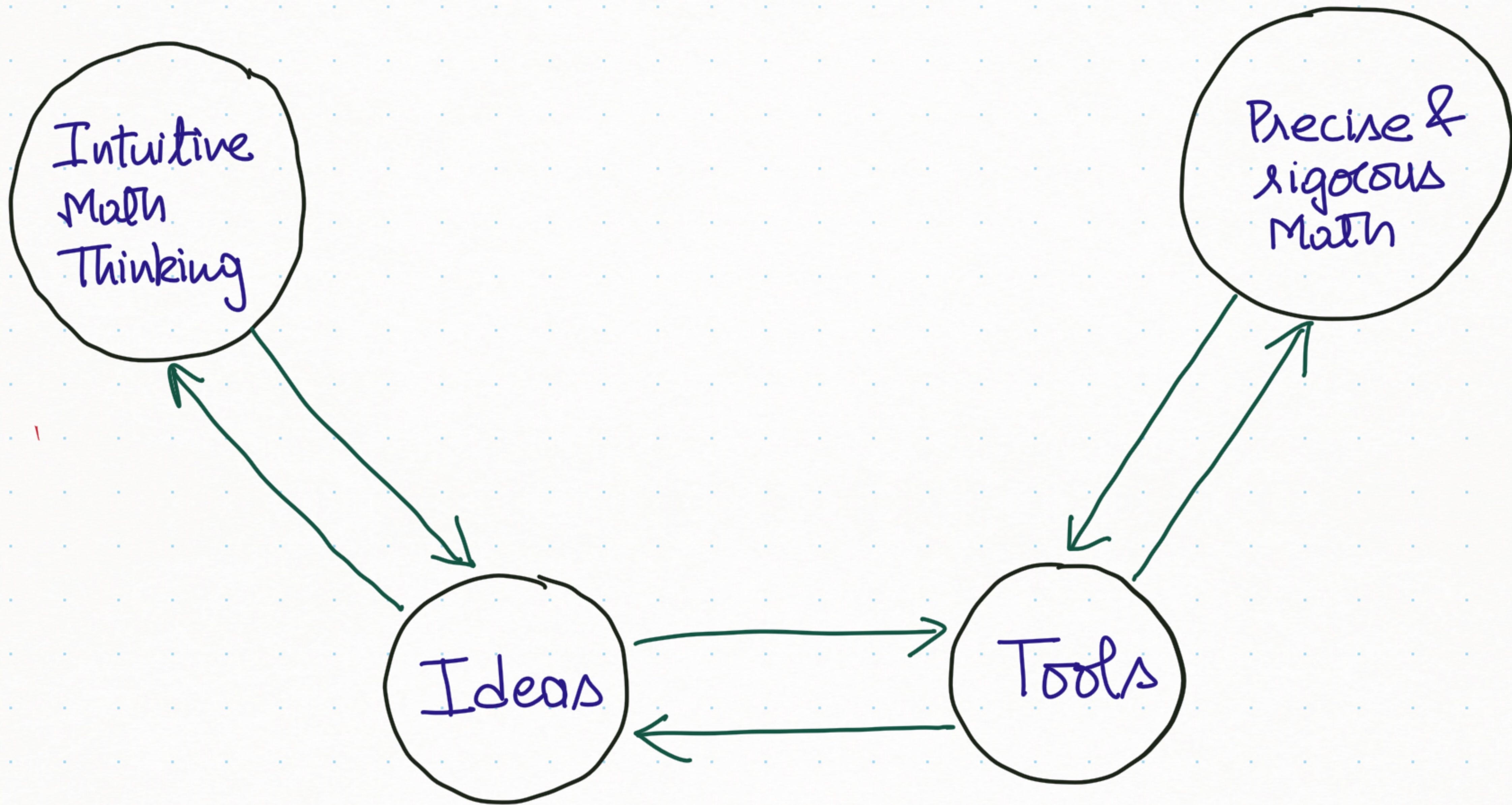
- A good model reveals relationships that may not be apparent superficially
- Mathematical analysis builds strategies/courses of action that are more sophisticated/powerful than a naive approach
- Allows for experimentation (simulation) when it's impossible or too expensive in the real world.

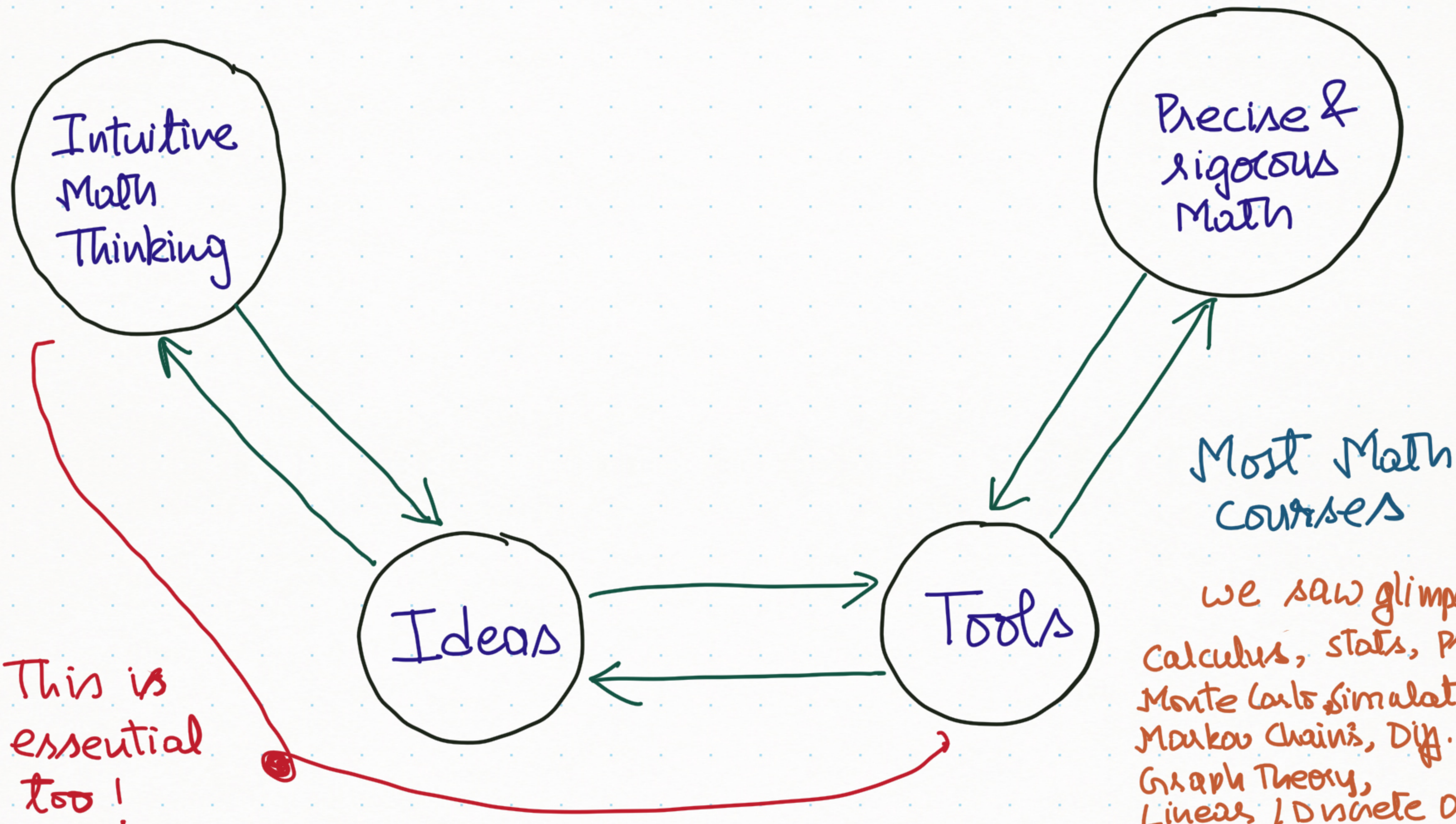
Skills

→ Mathematical knowledge & expertise

→ Computational experience

→ Effective communication to/in a group





This is essential too!

Most Math Courses

we saw glimpses of
Calculus, Stats, Probab,
Monte Carlo Simulation,
Markov Chains, Diff. Eqns,
Graph Theory,
Linear / Discrete Optimiz,
& more.

Intuitive
Math
Thinking

Build a bridge
In fact, a network
of connections

Precise &
rigorous
Math

Ideas

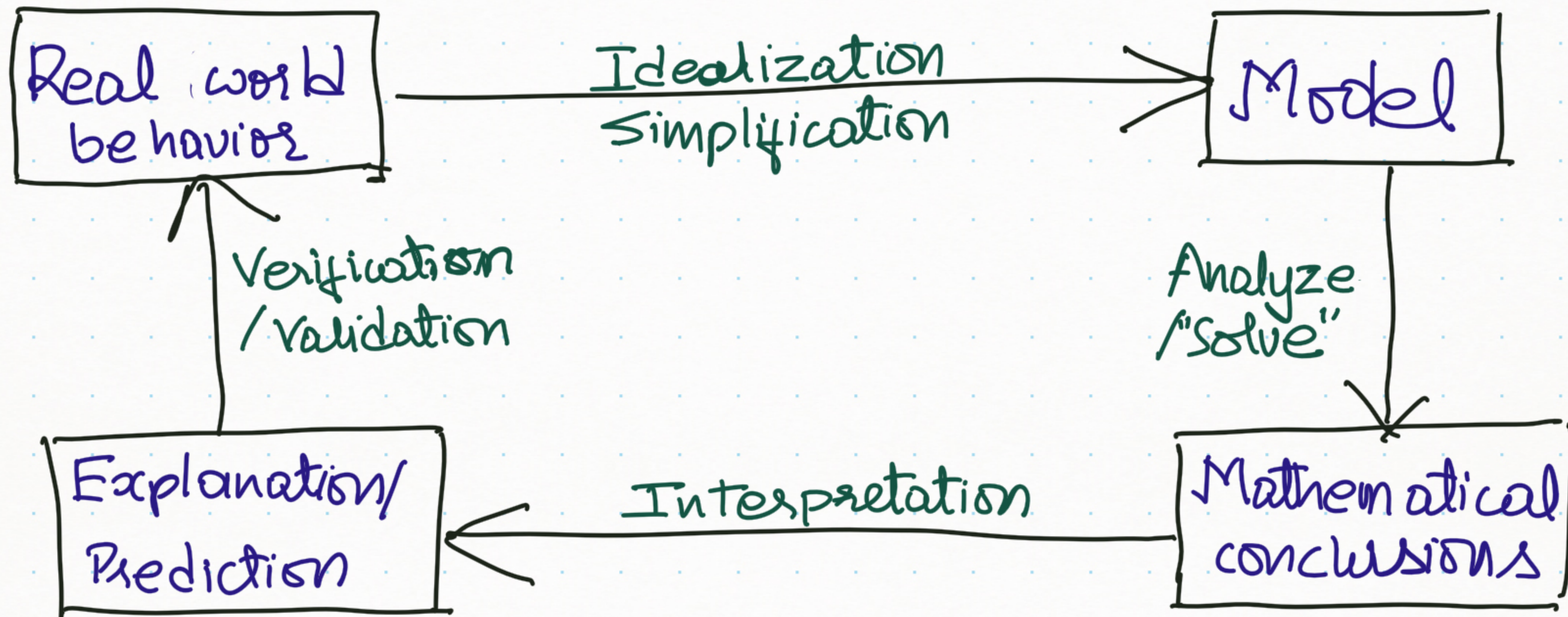
Tools

Most Math
Courses

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Calculus, Stats, Probab,
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& more.

This is
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Mathematical Modeling



Where are ethical concerns important above?

What are ethics?

Why are ethics important for Applied Mathematician?

What is Ethical Decision making?

What is Ethical Decision making?

Ethics is more than
or beyond:

- Feelings
- Religion
- Law
- Culture
- Science

How do we "judge" a decision/situation
in terms of Ethics?

How do we "judge" a decision / situation
in terms of Ethics?

— "morality"

— responsibility, towards
"common good"

— maximizing "happiness"

—
⋮

Some sources of Ethical Standards

Utility Which option will produce the most good and do the least harm?

Rights Which option best respects the rights of all who have a stake?

Fairness/Justice Which option treats people equally or equitably?

Common Good Which option best serves the community as a whole?

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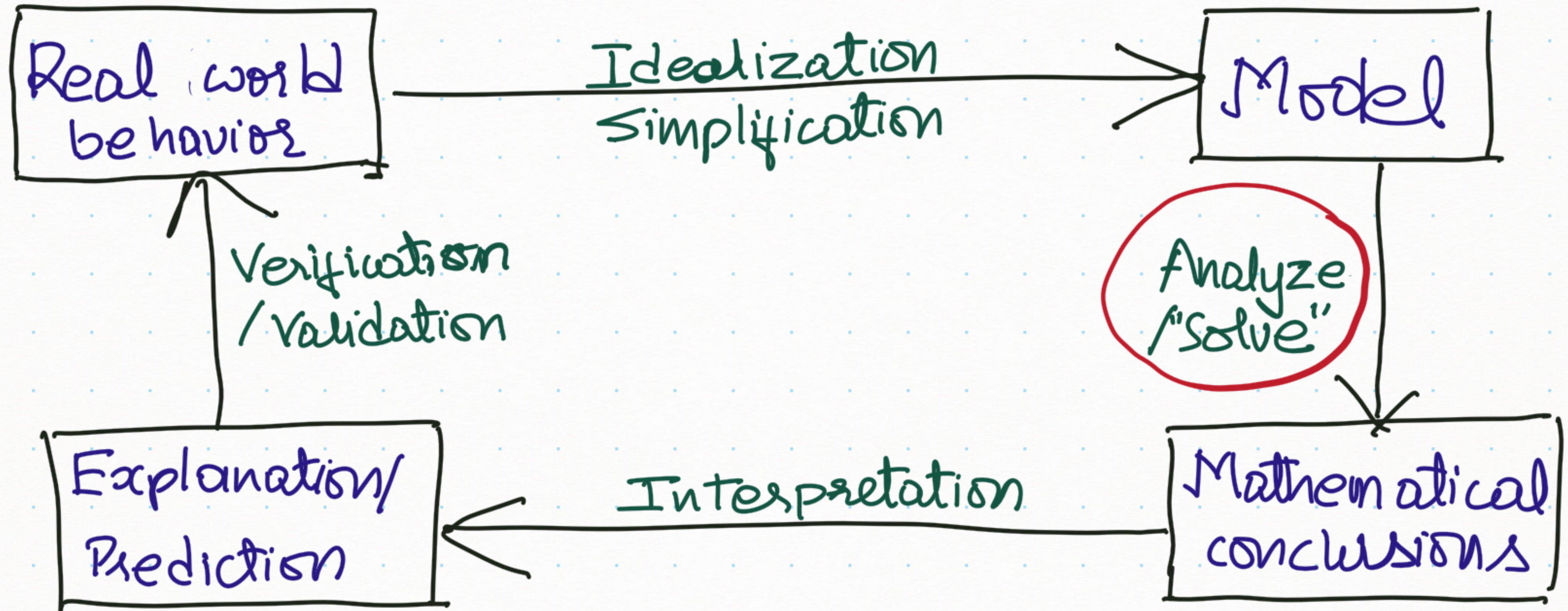
Virtue Which option leads me to act as the sort of person I want to be?

Look up

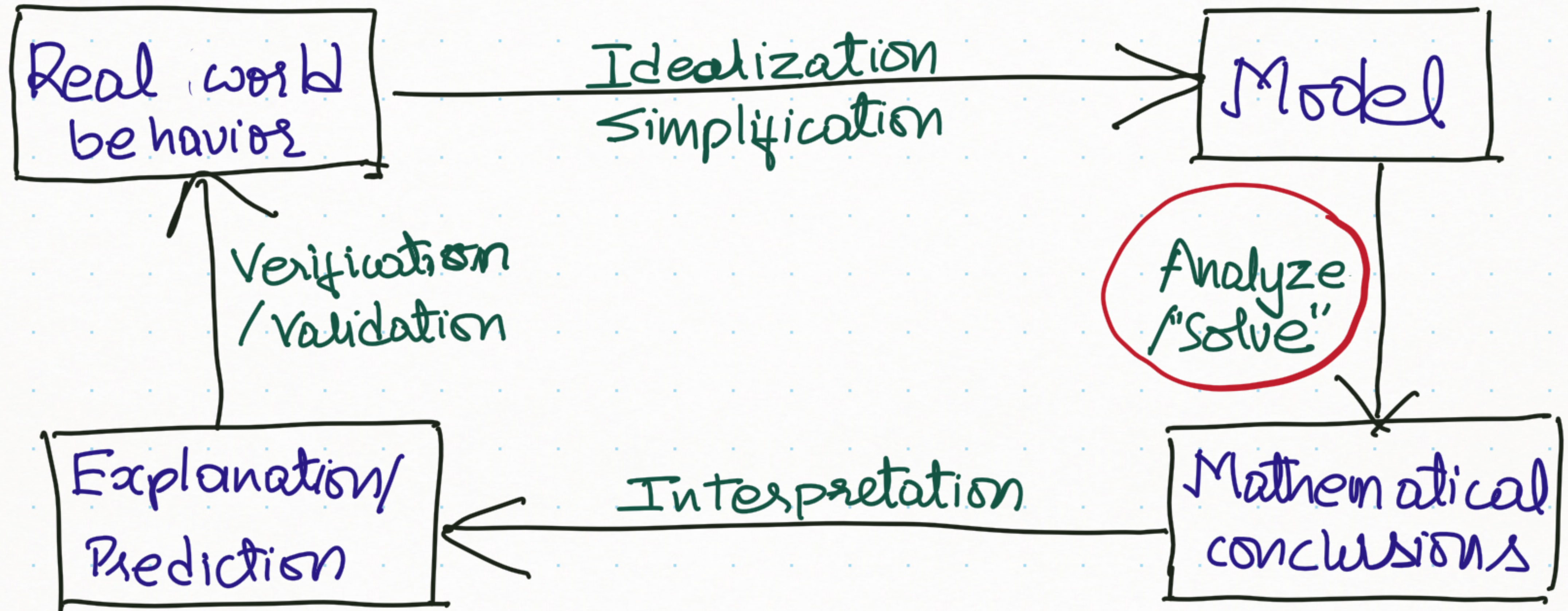
Marckula Center
for Applied Ethics

for more discussion
& resources.

Mathematical Modeling

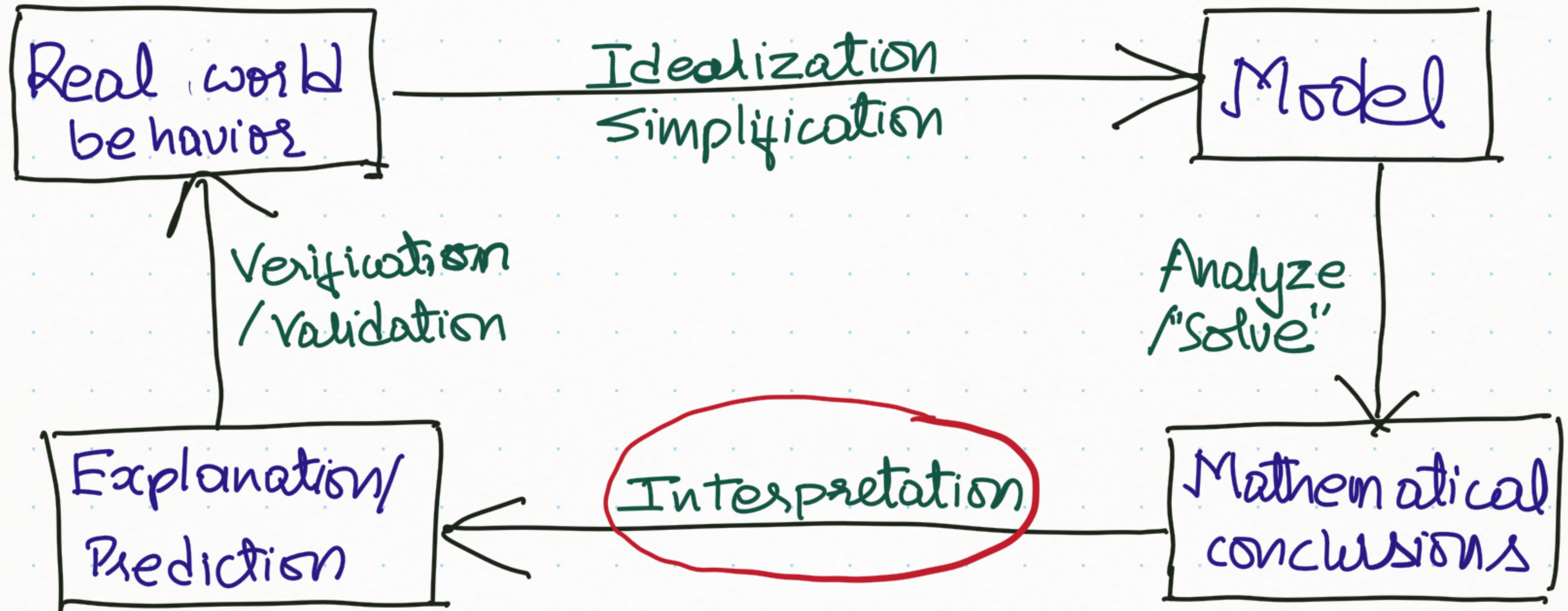


Mathematical Modeling

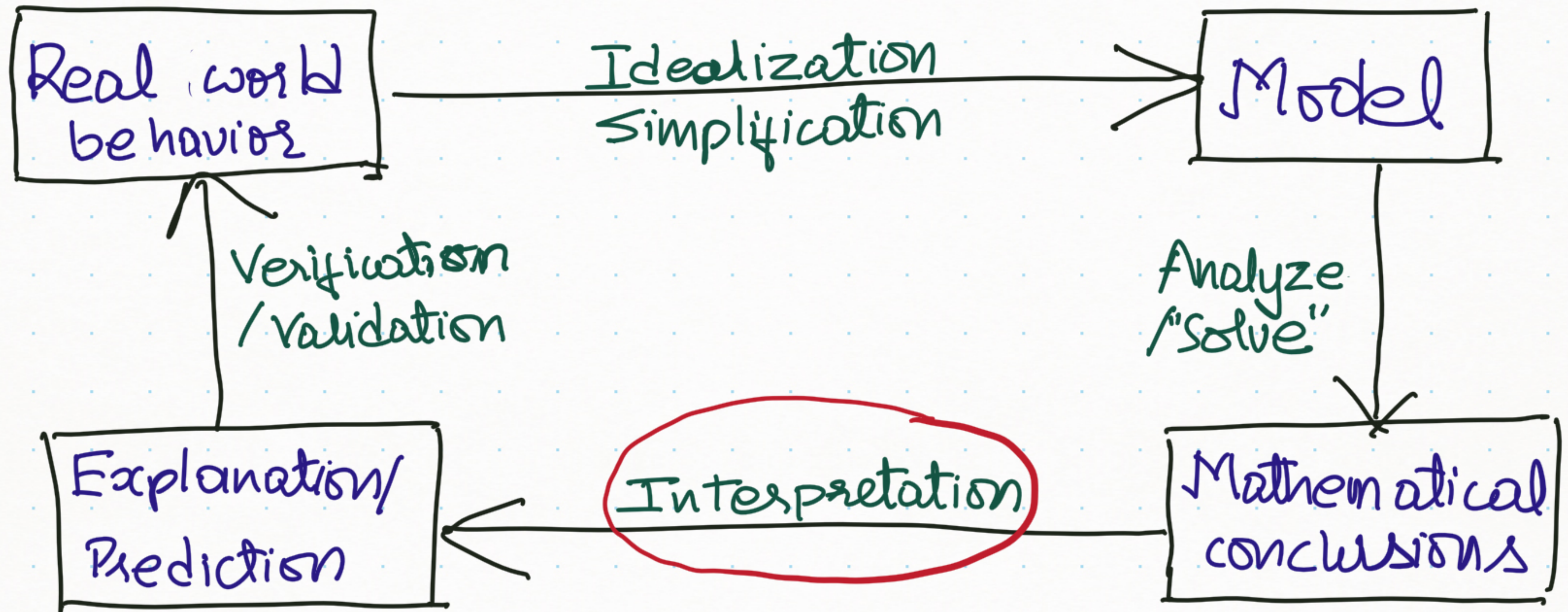


e.g. Algorithmic ethics

Mathematical Modeling

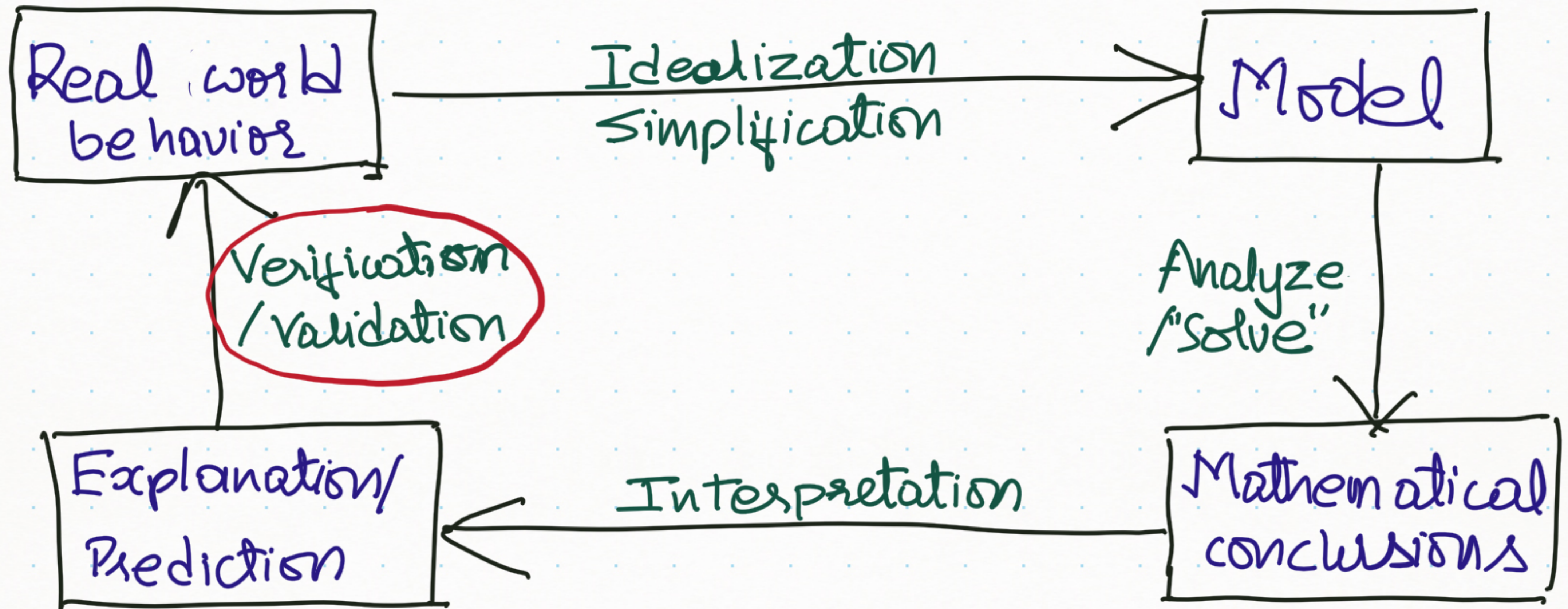


Mathematical Modeling

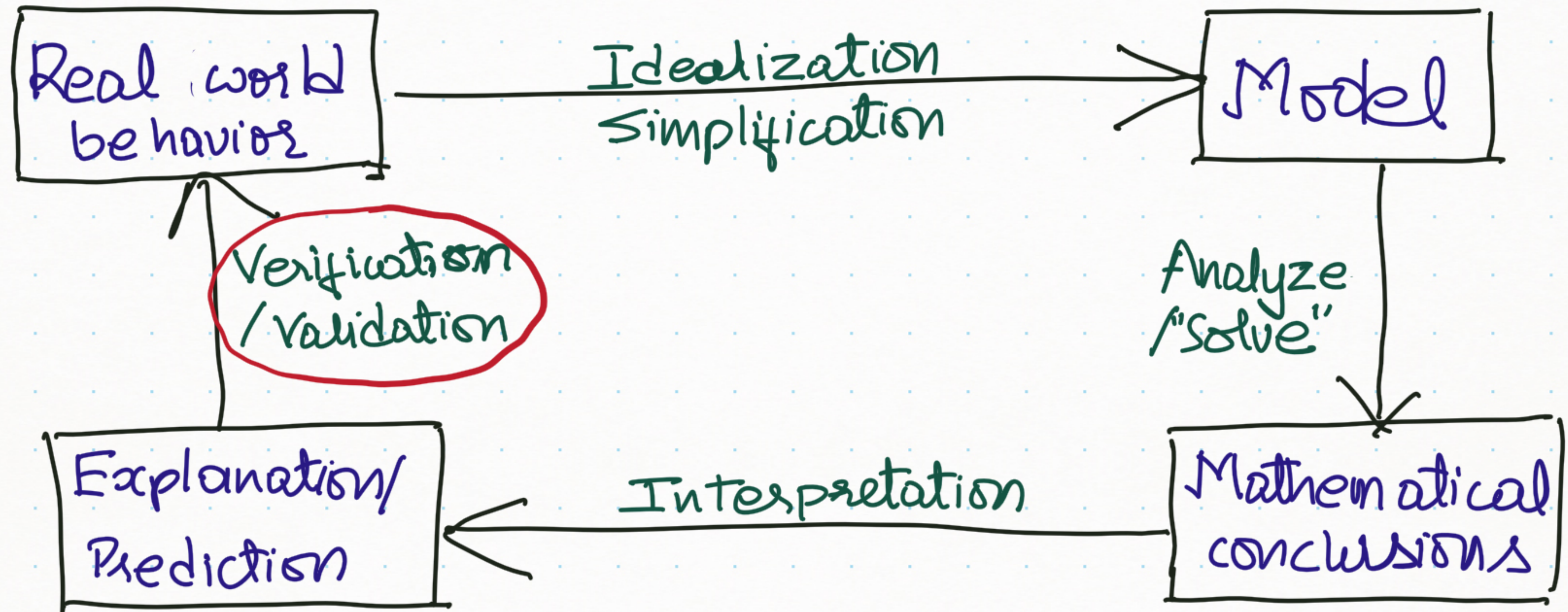


e.g. Bioethics

Mathematical Modeling

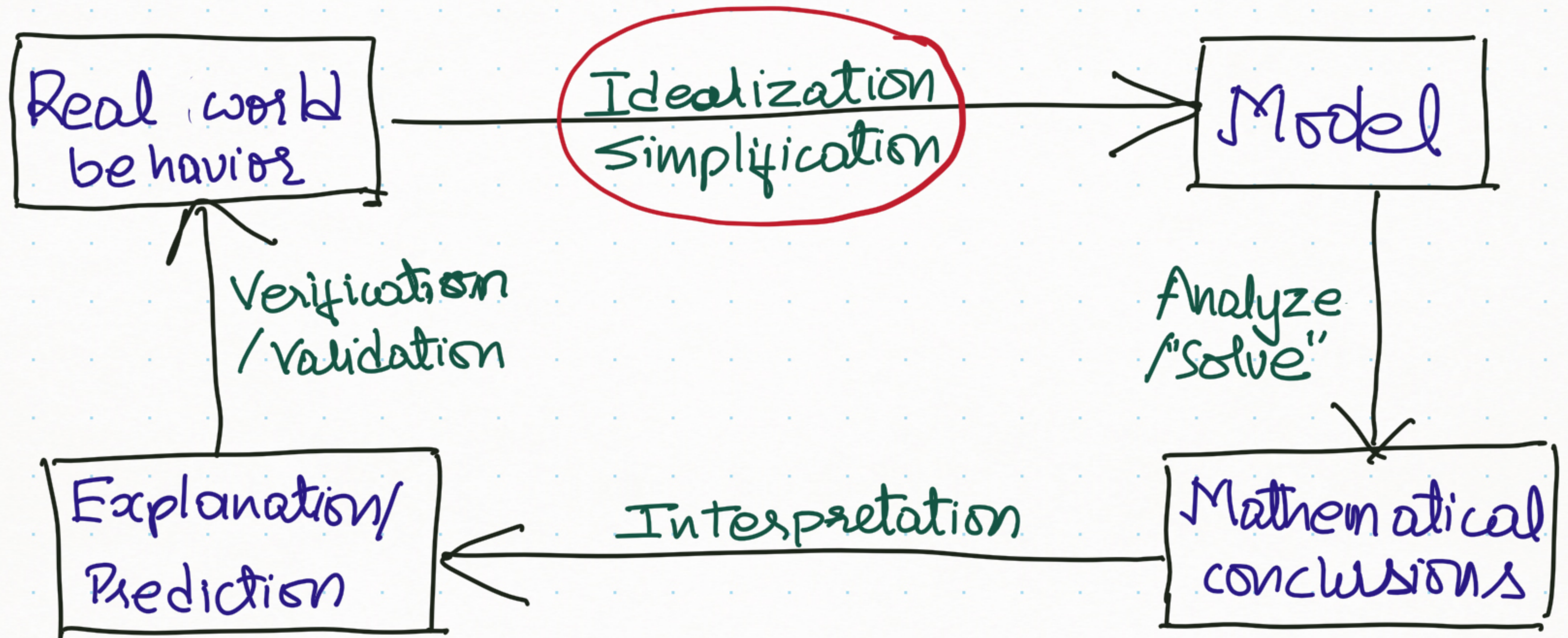


Mathematical Modeling



e.g. Statistical Ethics

Mathematical Modeling



Often ignored or assumed to be unbiased

Are our mathematical assumptions creating an inbuilt bias into our model that cannot be overcome even if we do everything else perfectly — avoid any bias in the algorithmic computation, the statistical / data collection and analysis, and in its real-world recommendations?

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→ Love Canal Controversy (1978 — 1980s)

Love Canal Controversy

- Love Canal, suburban town in NY state close to Niagara Falls
- 1890s a canal built by W.T. Love for an hydroelectric plant (never built)
- 1905 Hooker Electrochemicals factory established
- 1942-1952 HE allowed to dispose 22,000 tons of chemical waste in the canal in fiber & metal barrels.
- 1953 Canal was full, so covered with soil and grass
- 1955 school for 400 children built, followed by a small town.
- 1970s chemical odors noticeable
- 1978 Newspaper articles & NYS Dept. of Health investigation
80+ chemicals found in soil incl. 10 carcinogens

• 1978 Govt. offered to buy 239 houses closest to the waste site and relocate residents. Waste site sealed.

• 1978 Health department did detailed health check-ups of the residents.

Conclusion: Love Canal safe to live in.

• 1978 Lois Gibbs, a home maker, organized a Residents Assoc. She sat down with a map of LC & put a pin on every house with registered medical problem.

Noticed a pattern of narrow paths corresponding to filled in old streams and swales — the "wet homes".

• 1978 Lois Gibbs & Beverly Paigen (cancer researcher nearby) discovered women in 'wet homes' had 3 times more miscarriages & birth defects & ... than in 'dry homes'.

• 1978 Gibbs hypothesis confirmed by Paigen →

The strategy resident evacuation should be 'wet homes' first.

• Contrary to strategy by scientists at health dept. evacuation based on distance from the chemical waste site.

This was based on ^{unbiased} mathematically accurate analysis with the scientifically appropriate assumption that toxics spread radially from the chemical waste site.

Research group at D&H said "Investigation of Gibbs-Paigen was totally incorrect" & "The information collected by housewives is useless".

• 1981 With help of media coverage, Gibbs-Paigen prevailed & residents were all evacuated ...

The difference between the approaches of Doh scientists and Paigen

- Doh scientists assumed toxins spread more or less homogeneously outward from the waste site.

This is traditional scientific practice (going back to Galileo) to make the basic simplified model, to which more complexities could be added later.

Paigen made the assumption that toxins spread along swales.

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Scientists → Need ^{strong} evidence to conclude LC is unsafe.
↳ Objective "we deal only with numbers, we are scientists".

Paigen → Need strong evidence to conclude LC is safe
since a mistake could result in damage to human life.

↳ "question is not whether you can publish this research in New England Journal of Medicine but would you let your daughters work there?"

Precautionary Principle

Paigen's position was an application of what has become known as "The Precautionary Principle".

Originated in 1970s, formalized in 1992 under the Rio Declaration (principle 15):

"Where there are threats of serious or irreversible damage, lack of full scientific certainty shall not be used as a reason for postponing cost-effective measures to prevent environmental degradation."

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Numbers, Math, Models can be wrong even if the basic assumptions are scientifically accepted & applied correctly

1986, a cloud of radioactive material from Chernobyl passed over Cumbria, North Wales, UK. Heavy rains caused radioactive Cesium to fall over grazing lands for sheep.

- "No need to worry" radioactive material will disperse soon. High radioactivity after 6 months, ban on sheep meat, continued on for 6 years in some areas. Why?

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Scientific Assumption (verified locally): Cesium would sink far into ground after a short period of time. Verified in the alkaline clay soil.

(underlying assumption: radiation would come from Cesium in soil) similar to alkaline peat soil as a physical transport ←

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- "No need to worry" radioactive material will disperse soon. High radioactivity after 6 months, ban on sheep meat, continued on for 6 years in some areas. Why?

• Reality: Sheep received cesium in their bodies through grass.

Soil as a physical vs. chemical transport medium:

Clay → cesium adsorbs on aluminium silicate & becomes inactive

Peat → cesium remains chemically active.

Original scientists were Physicists & not Chemists.

- When doing applied math / science, mathematicians / scientists are not "outside the system".
Scientific objectivity is tricky.
- Uncertainty of assumptions & evidence must ^{be} considered & be careful about "burden of proof".
- Laypeople / local stakeholders should ^{be} involved, encourage them to ask "stupid" questions that poke holes in underlying assumptions.

How can ethical concerns guide applied math?

How should a public transportation be designed?

Standard objectives:

- min operator costs
- min user costs

subject to

standard transportation
network design and flow
constraints

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How should a public transportation be designed?

Standard objectives:

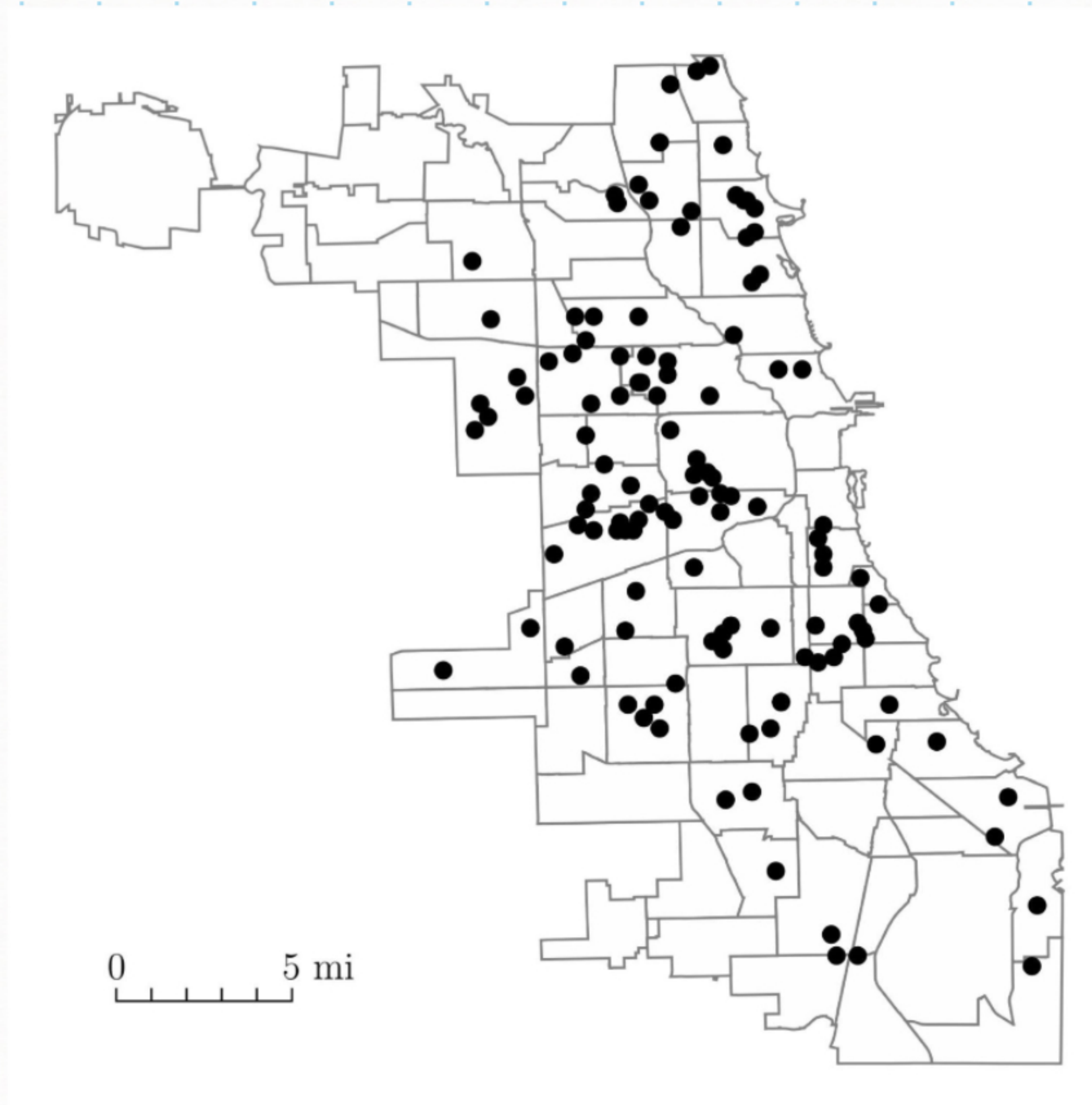
min	operator costs	} economic costs combined together
min	user costs	

subject to

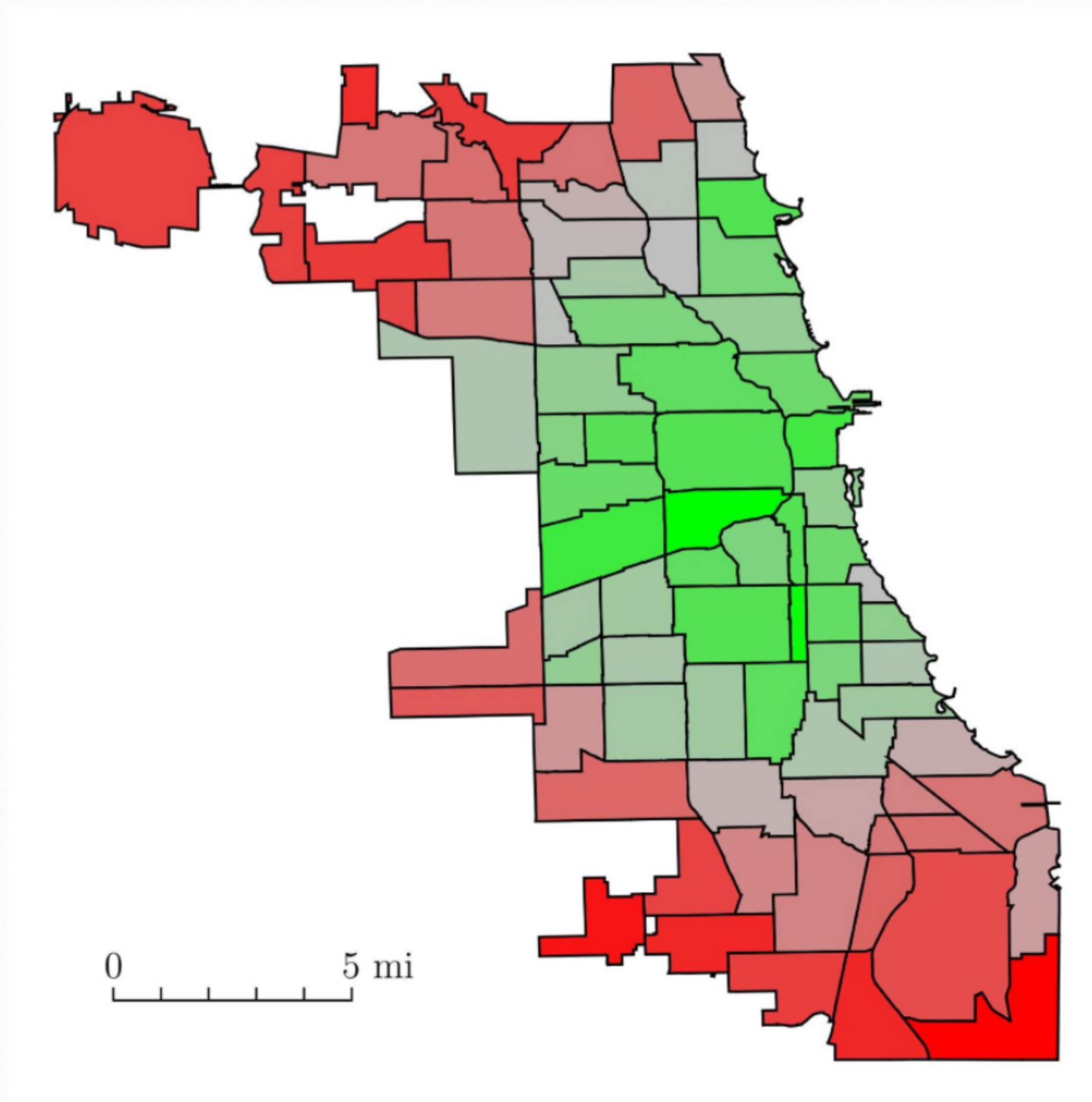
standard transportation network design and flow constraints

But what about accessibility to social services?

Do all communities have equitable ^{transportation} access to museums, libraries, parks, city offices, healthcare centres, etc.?



Primary Healthcare community centers in Chicago



Accessibility metric value of each of the 77 community areas in Chicago w.r.t. primary healthcare centers.

How can we improve accessibility / make it fair across all communities?

- Build more CTA subway lines
- Add more CTA bus lines
- Build more primary care centers

} Need big financial commitments from Govt. ↗ unlikely!
↖ slow!

How can we improve accessibility / make it fair across all communities without worsening overall performance?

maximize accessibility metric across all communities

subject to

user costs $\leq (1+\epsilon)$ min user costs

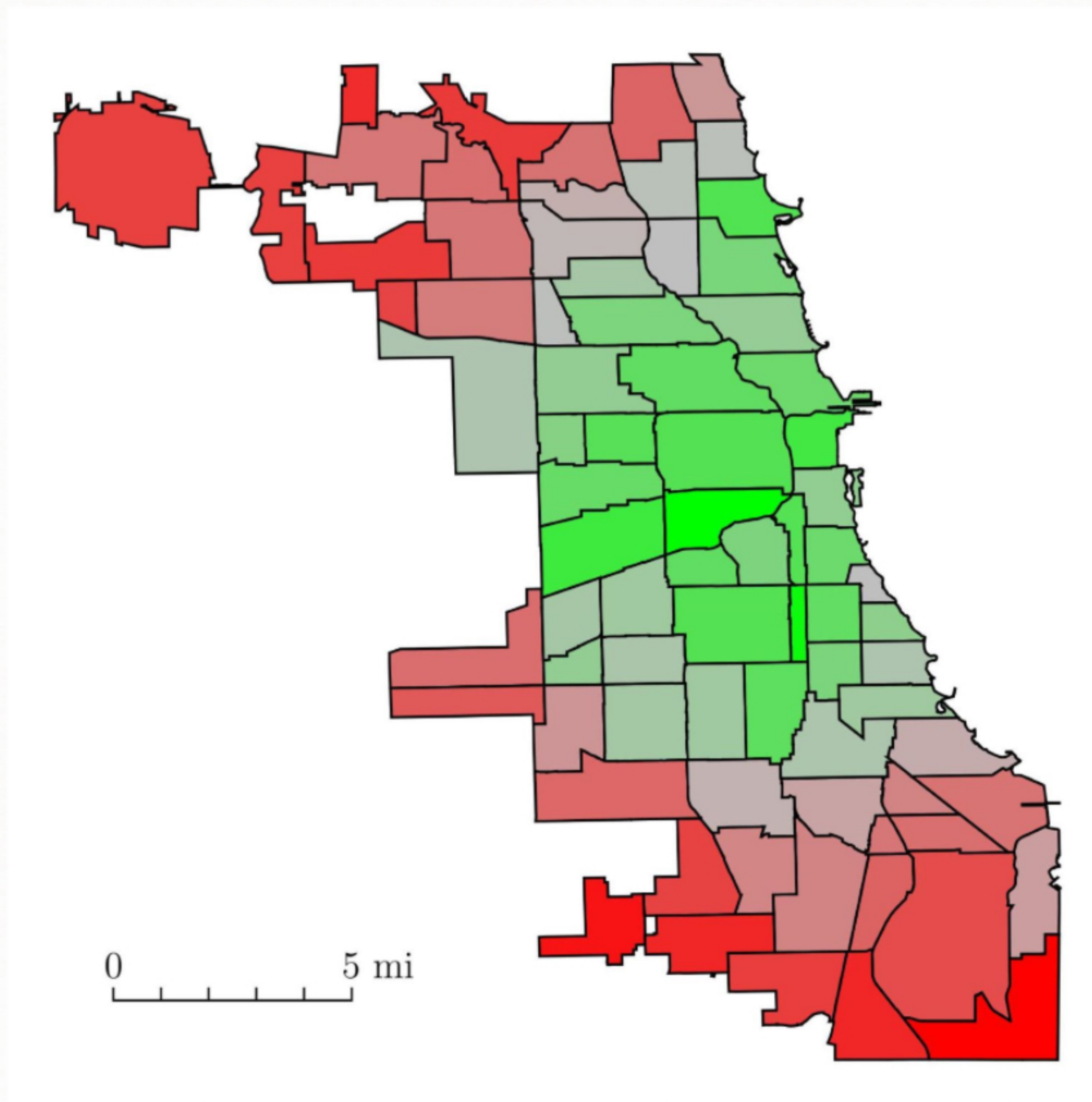
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transportation network constraints

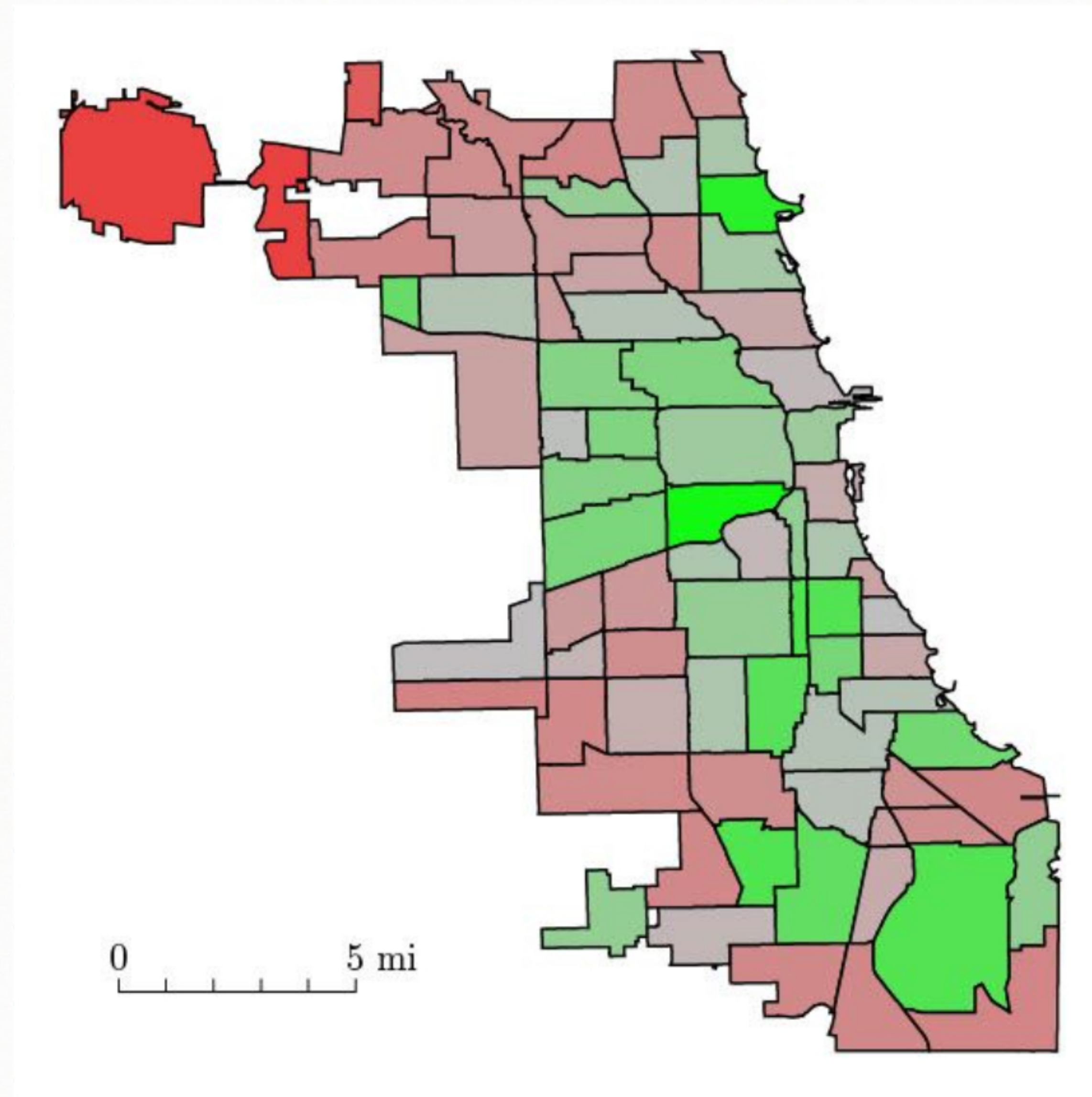
with decision variables for \rightarrow change of bus frequencies

\rightarrow interchange of buses between slow & express

while maintaining same number of buses.



Accessibility metric value of each of the 77 community areas in Chicago w.r.t. primary healthcare centers.



Improved accessibility metric values after implementation of new model for equitable access to social services.