MATH 380

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Drug Dosage A patient is prescribed 250 mg of a drug every 4 hours. 30% of the drug in the bloodstream is eliminated by the patients body every 4 hours.
How much drug will be in the patient's bloodstream after 72 hours? Long term?

Step 1

Step 2

Step 3

Step 4

Step 5

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Step 1 Identify the problem

Step 2 Assumptions/simplifications, & variables, etc.

Step 3 Construct the model

Step 4 Solve & interpret the model

Step 5 Validate the prediction vs. real data/observations

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time? n = number of 4-hour time periods. $0, 1, 2, \ldots$

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time? n = number of 4-hour time periods. 0, 1, 2, ...

drug? $a(n)$ = amount of drug in the bloodstream after period n, $n = 0, 1, 2, \ldots$

assumptions?
- patient does not have any abnormalities
- no other drugs / interactions in the bloodstream
- no internal / external factors that affect drug absorption
- patient takes the drug at the correct time with correct dosage
  - drug is immediately ingested into the bloodstream

Step3 The Model

Change = dose − loss from the system

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i.e., $a(n+1) - a(n) = 250 - (0.3)\,a(n)$

i.e., $\boxed{a(n+1) = (0.7)a(n) + 250}$

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After 72 hours:

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It can be solved exactly as: $\boxed{a(n) = \frac{2500}{3} - \frac{2500}{3}(0.7)^n}$

After 72 hours: $a(18) = 831.98\,mg$

Long term: $\lim_{n\to\infty} a(n) = \frac{2500}{3} = 833.33\,mg$

Is this acceptable?

Solutions to Dynamical Systems

Method of conjecture Look for a pattern;
Conjecture;
test the conjecture & reach a conclusion

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e.g. Recall savings certificate example.

$a_{n+1} = (1.01)\,a_n$ with $a_0 = 1000$ ← Formula for a_n?

Pattern
$a_1 = (1.01)\,a_0$
$a_2 = (1.01)\,a_1 = (1.01)(1.01\,a_0) = (1.01)^2 a_0$
$a_3 = (1.01)\,a_2 = (1.01)(1.01)^2 a_0 = (1.01)^3 a_0$
$\vdots$

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Conjecture $a_n = (1.01)^n a_0$

Test $a_{n+1} = (1.01)\, a_n = (1.01)(1.01)^n a_0 = (1.01)^{n+1} a_n$

$$\boxed{\begin{aligned} a_n &= (1.01)^n a_0 \\ &= 1000\,(1.01)^n, \quad n \geq 0 \end{aligned}}$$

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Long-term behavior

r	Behavior of a_n
$r=0$	
$r=1$	

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$r<0$	a_n oscillates
$\lvert r \rvert < 1$	a_n converges to 0
$\lvert r \rvert > 1$	a_n diverges

Defn A constant C is called an equillibrium value of a DDS $a_{n+1} = f(a_n)$ if $a_n = C$ for all $n \geq 1$ when $a_0 = C$.

"Starting from $a_0 = c$ traps the DDS at $a_n = c$"

That is $a_{n+1} = f(a_n)$ becomes $C = f(C)$ when $a_0 = C$

"Fixed Point"

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example Recall the drug dosage problem

$a(n+1) = (0.7)a(n) + 250$

If c is an equillibrium value Then $c = (0.7)c + 250$

i.e., $(0.3)c = 250$

i.e., $c = \frac{250}{0.3} = \frac{2500}{3} = 833.3$

Verify $c = 833.3\overline{3}$ is an eq. value

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Verify $c = 833.33$ is an eq. value

What does it mean that 833.33 mg is the equillibrium value?

<u>example</u> Recall the savings certificate problem.

$$a_{n+1} = (1.01)\,a_n - 50$$

[1% per month interest
$50 per month withdrawal]

Equillibrium value?

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Equillibrium value?

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What equillibrium value of $5000 mean here?

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Equillibrium value?

$c = (1.01)\,c - 50 \iff (0.01)c = 50$ i.e., $c = \dfrac{50}{0.01} = 5000$

What equillibrium value of \$5000 mean here?

In order to withdraw \$50 per month indefinitely without changing the savings balance, we should make the initial deposit of \$5000.

Stable & Unstable Equilibrium values

e.g. $a_{n+1} = (0.5)a_n + 0.1$

Equilibrium? $c = 0.5c + 0.1$ gives $c = 0.2$

& we can verify setting $a_0 = 0.2$

gives $a_1 = 0.2, a_2 = 0.2, \cdots$

$\cdots, a_n = 0.2$

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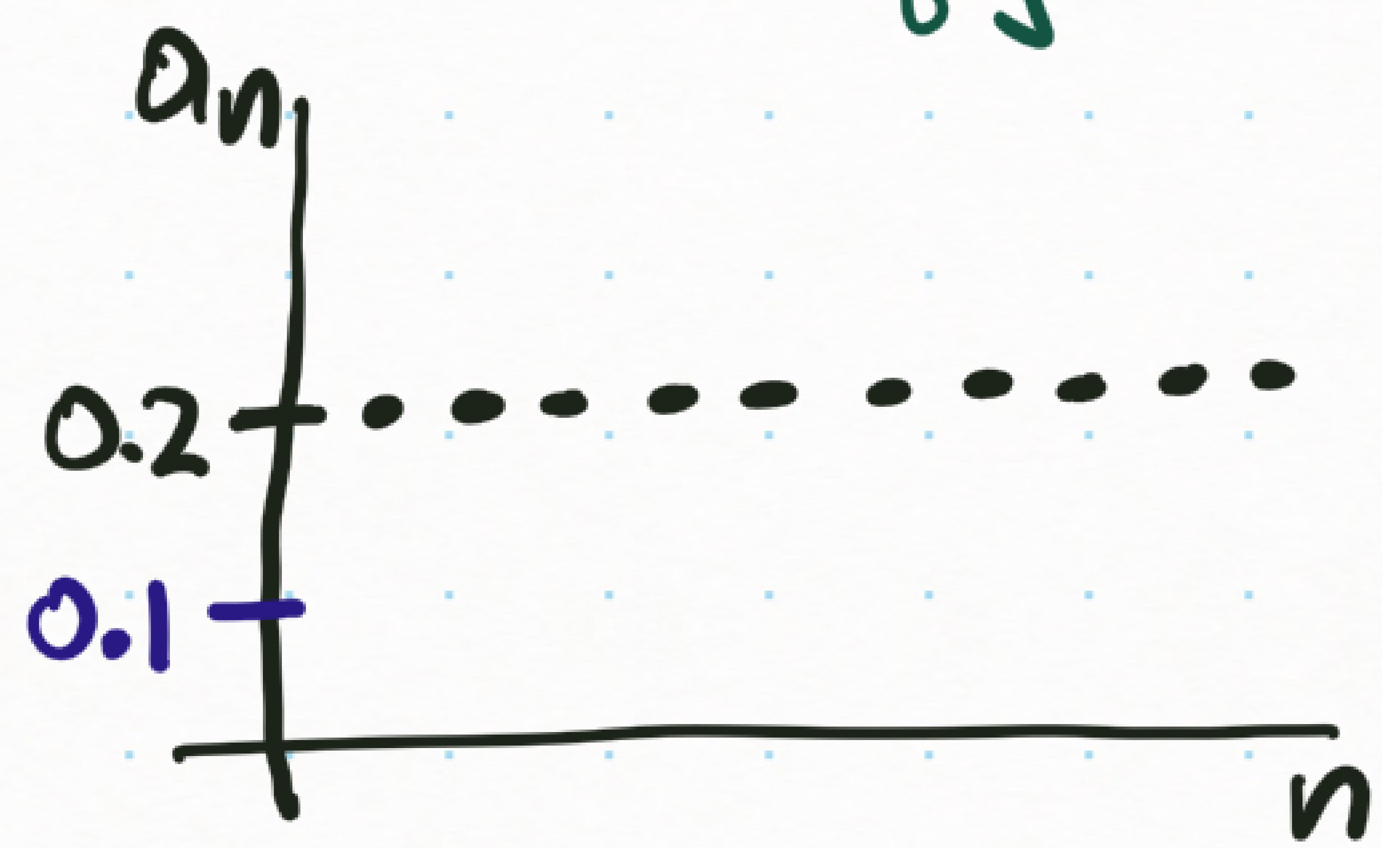

What if $a_0 < 0.2$?

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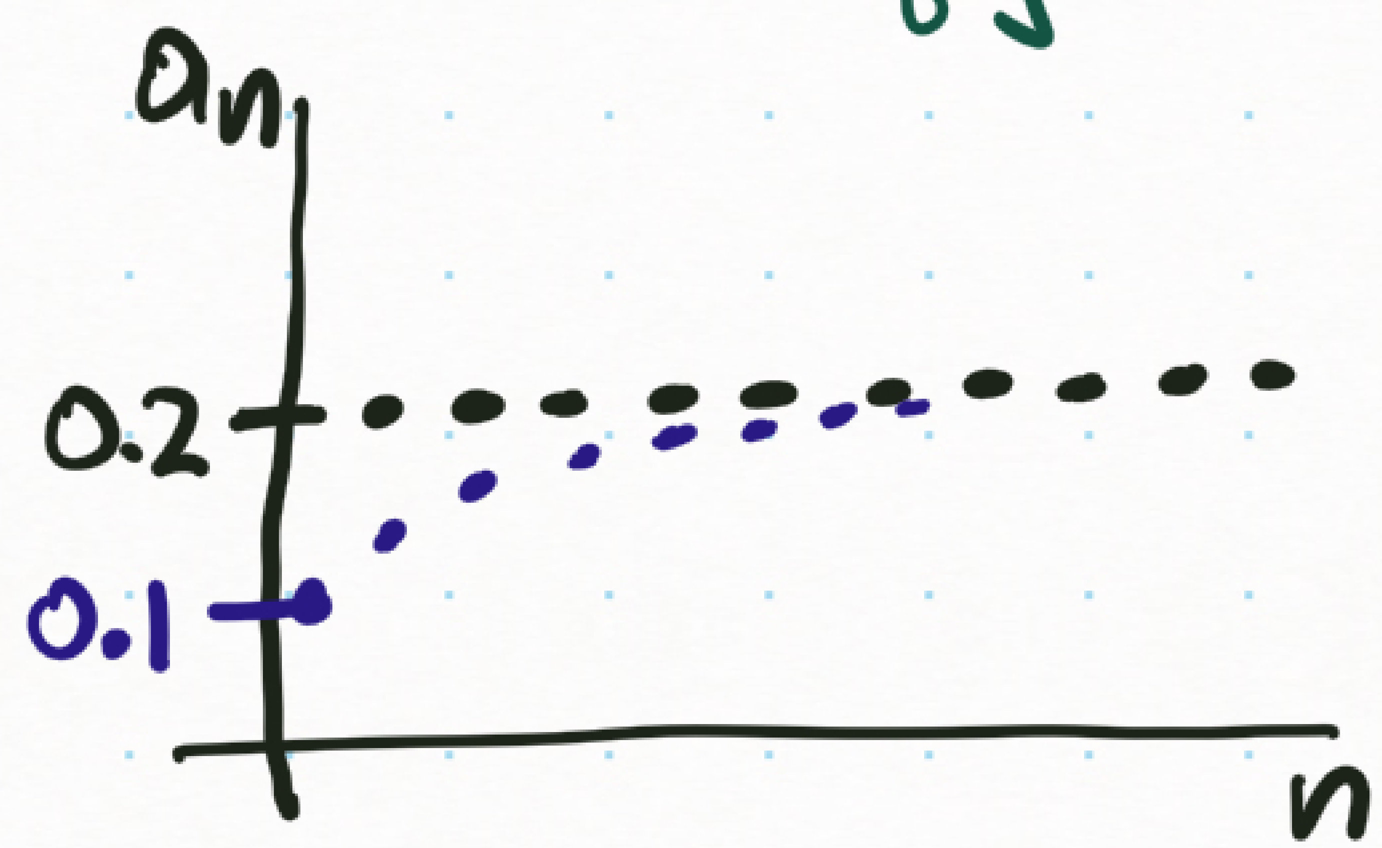

What if $a_0 < 0.2$?

say $a_0 = 0.1$

$a_1 = 0.15, \; a_2 = 0.175,$

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$a_{15} = 0.19999695,$

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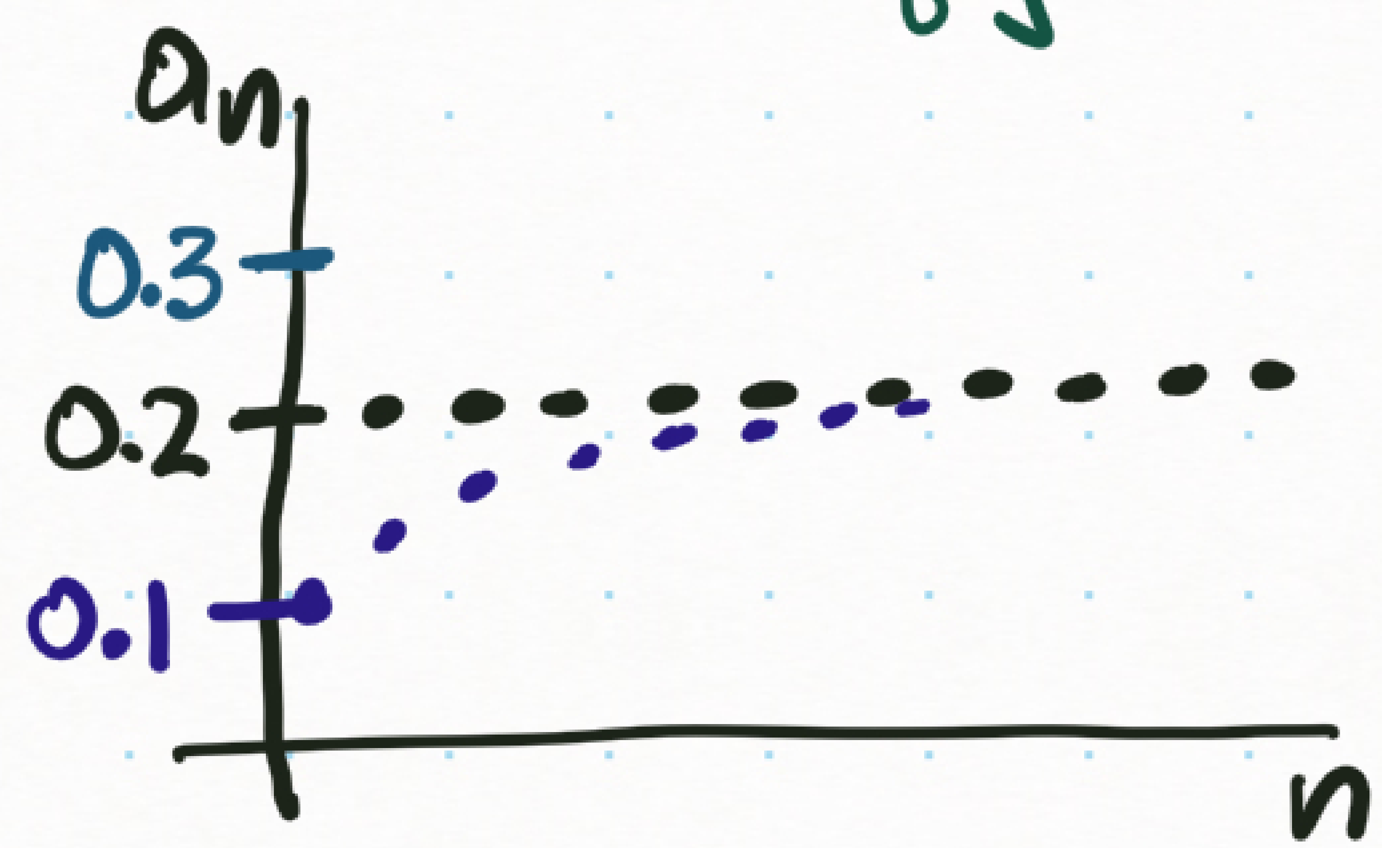

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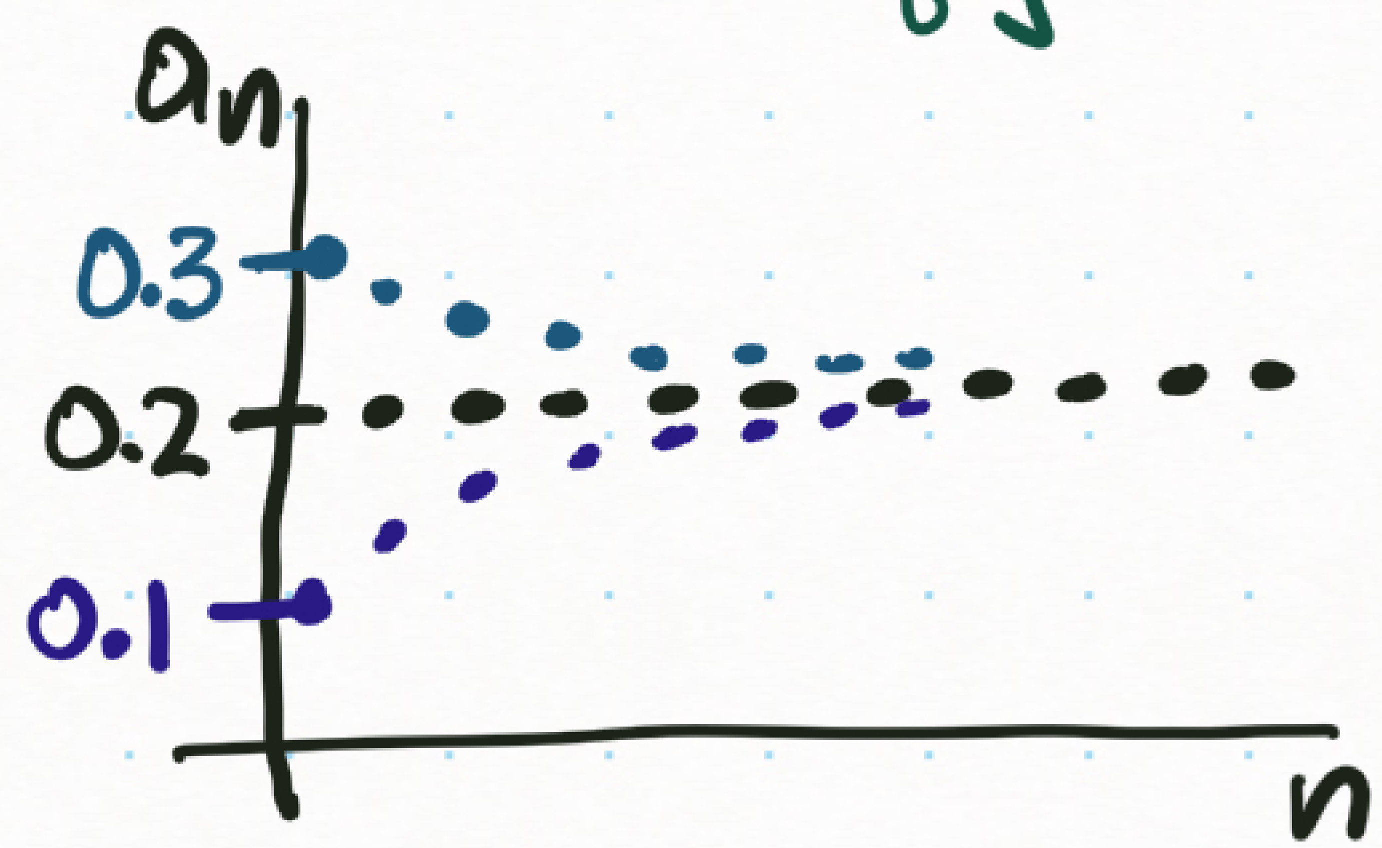

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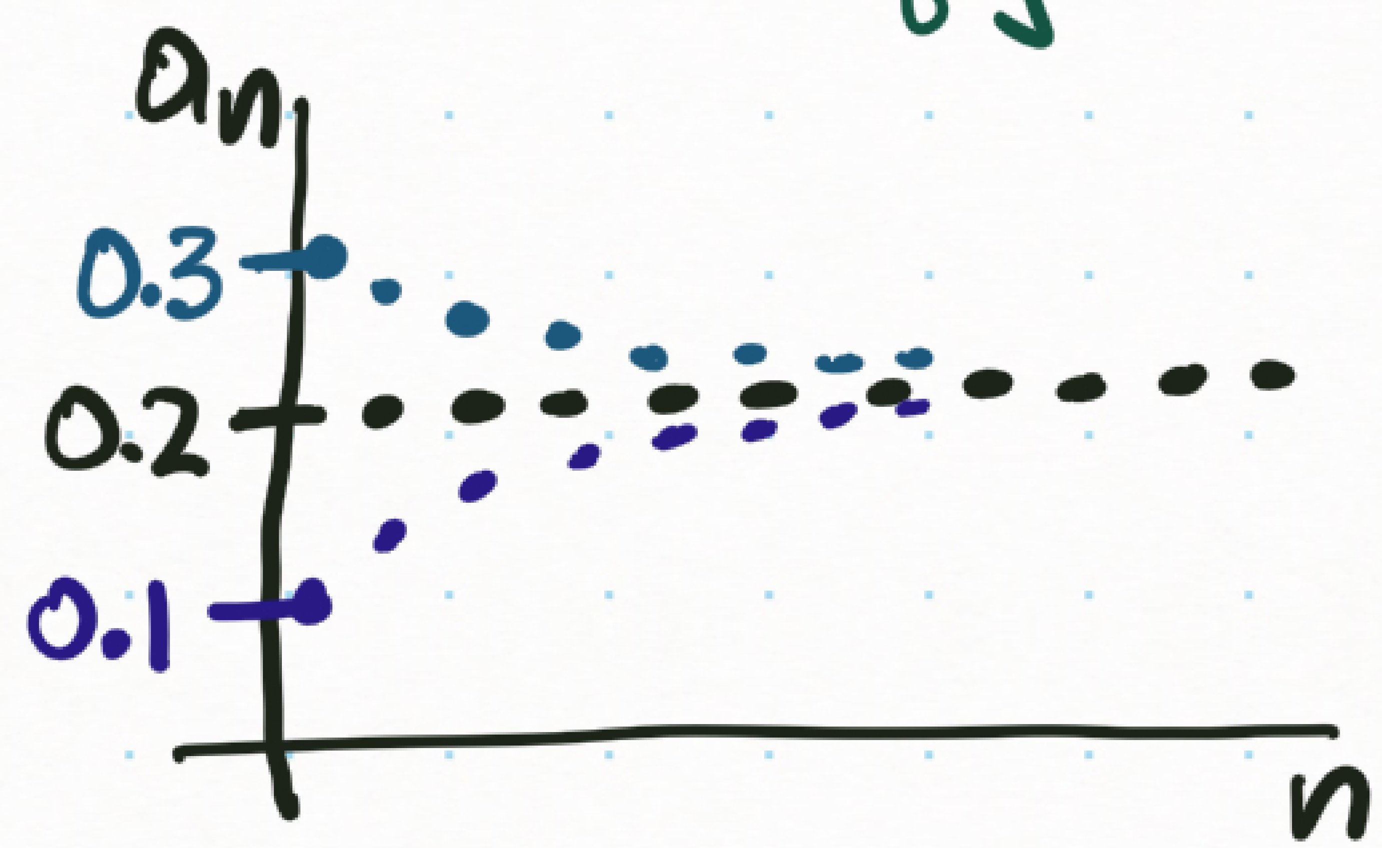

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say $a_0 = 0.3$

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We say $c = 0.2$ is a stable equilibrium

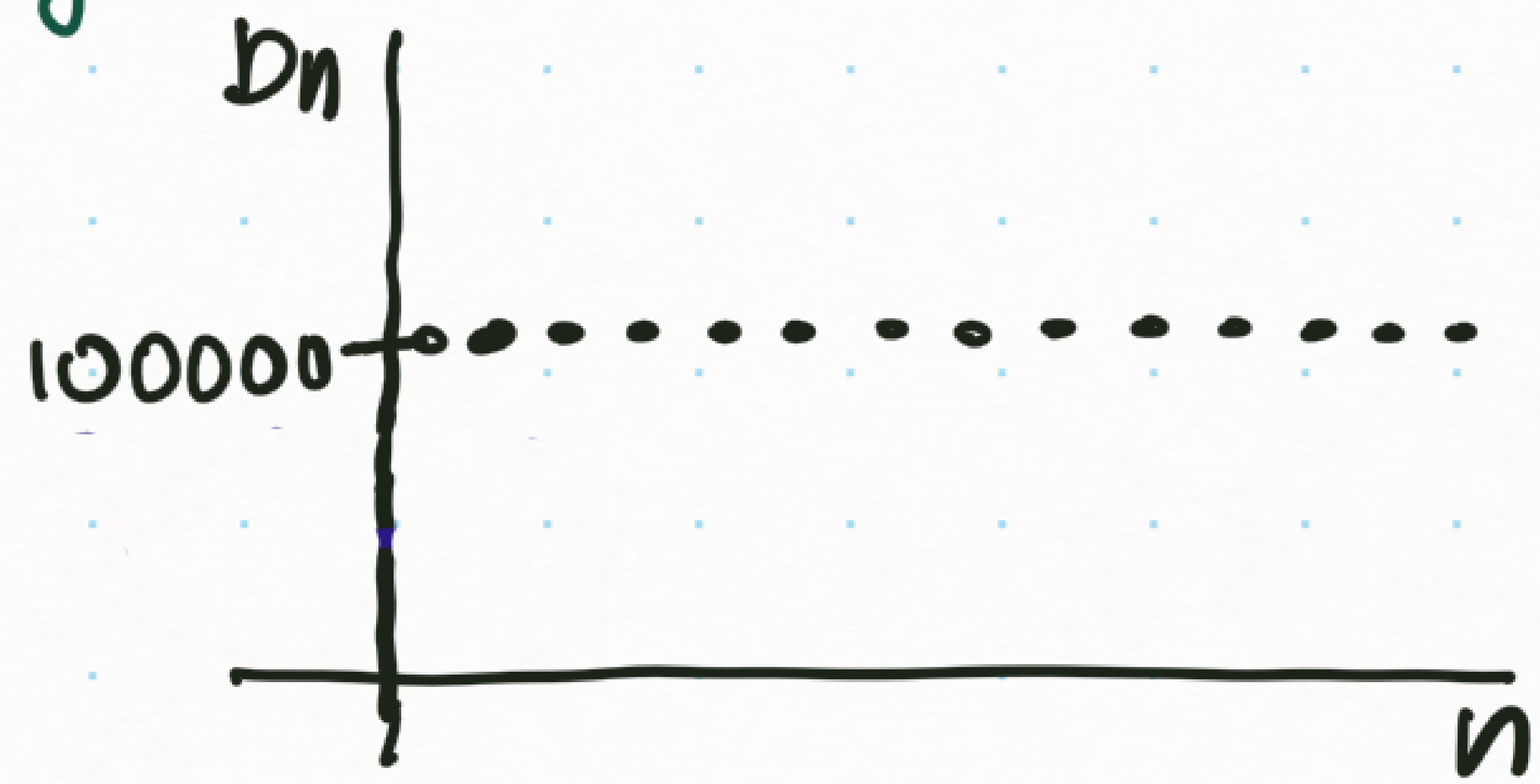
Stable and Unstable Equilibrium values

e.g. $b_{n+1} = (1.01)\, b_n - 1000$

Equilibrium? $c = (1.01)c - 1000$, ie, $c = 100000$

& we can verify setting $b_0 = 100000$

gives $b_1 = b_2 = \cdots = 100000$



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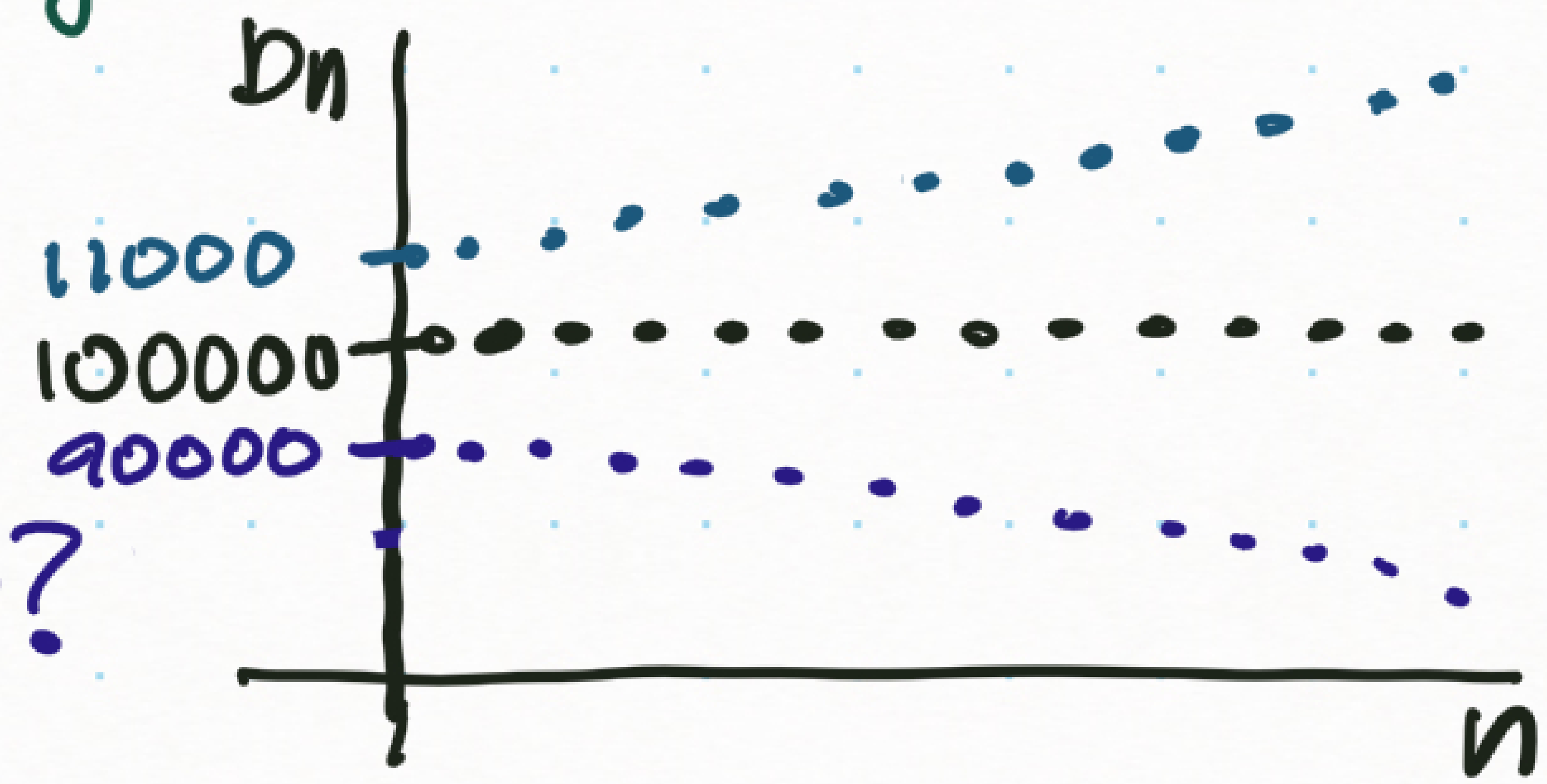

What if $b_0 < 100000$?

say, $b_0 = 90000$.

$b_1 = 89900$, $b_2 = 89799$,

$\ldots$, $b_{15} = 88390, \ldots$

What if $b_0 > 100000$?

say, $b_0 = 110000$

$b_1 = 110100$, $b_2 = 110201$,

$\ldots\ldots$, $b_{15} = 111610, \ldots$

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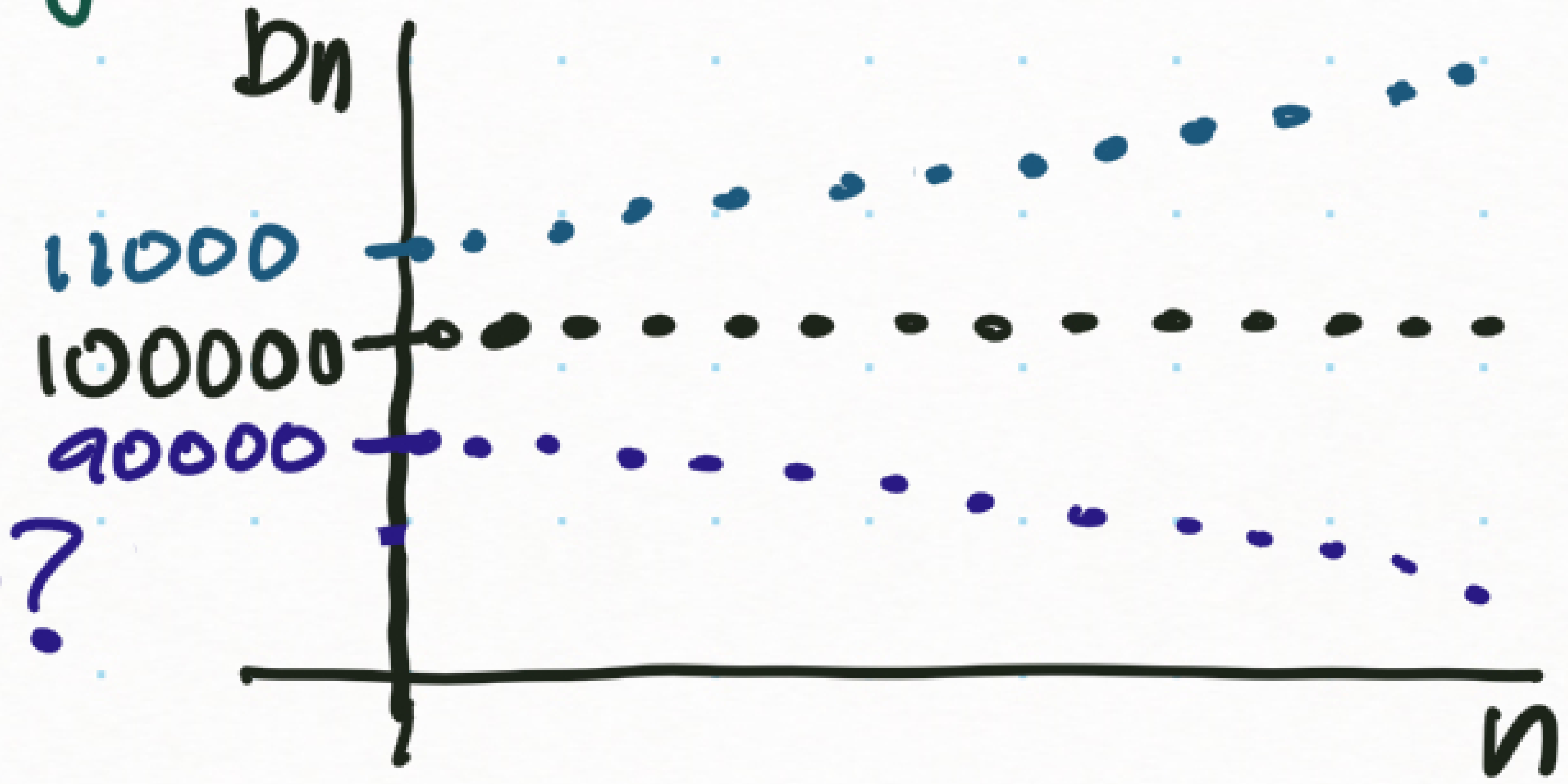

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What if $b_0 > 100000$?

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$b_1 = 110100$, $b_2 = 110201$,

$\ldots\ldots$, $b_5 = 111810, \ldots$

We say $c = 100000$ is an <u>unstable equilibrium</u>

Non-Homogeneous linear DDS

$$\boxed{a_{n+1} = r a_n + b}$$

Suppose $c \neq 0$ is an eq. value
then $c = rc + b$ i.e., $c = \frac{b}{1-r}$, $(r \neq 1)$

Theorem If $a_{n+1} = r a_n + b$ has a nonzero equilibrium value c then $c = b/_{1-r}$ when $r \neq 1$

when $r = 1$ & $b = 0$ then

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when $r = 1$ & $b = 0$ then every number is an eq. value.

when $r = 1$ & $b \neq 0$ then there is no eq. value.

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$\lvert r \rvert < 1$	
$\lvert r \rvert > 1$	

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r	long-term behavior
$\lvert r \rvert < 1$	Stable eq. value
$\lvert r \rvert > 1$	Unstable eq. value

For linear DDS, we can find explicit formulas, explicit long term behavior, etc.

But in general, for all sorts of DDS — — nonlinear, systems, etc. — we work with them experimentally.

Tables and plots of their values generated numerically using a simple program are used to analze their behavior both quantitatively and qualitatively.

Interacting Discrete Dynamical Systems

(That is, a system of Difference equations)

Competitive Hunter Model

Scarlet?

A habitat contains both spotted Owls and hawks.
They compete with each other for survival.
In absence of the other species,

When both species are present,

Build a model to understand the dynamics of both populations.

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A habitat contains both spotted Owls and hawks.
They compete with each other for survival.
In absence of the other species, each individual species shows unconstrained growth in population.
When both species are present, each population is affected negatively.

Build a model to understand the dynamics of both populations.

Let n = units of time period (say, 1 day)

Let O_n = size of owl population at the end of day n

H_n = —"— hawk ————————— " —————————

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Assumption change in population is proportional to current popula.

$\Delta O_n \propto O_n$ and $\Delta H_n \propto H_n$

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$\Delta O_n \propto O_n$ and $\Delta H_n \propto H_n$ ie, $\Delta O_n = k_1 O_n$ and $\Delta H_n = k_2 H_n$

where k_1, k_2 are constants

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$$\Delta O_n = k_1 O_n - k_3 O_n H_n \quad \text{and} \quad \Delta H_n = k_2 H_n - k_4 O_n H_n$$

where k_1, k_2, k_3, k_4 +ve constants.

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i.e., $O_{n+1} = (1+k_1) O_n - k_3 O_n H_n$ & $H_{n+1} = (1+k_2) H_n - k_4 O_n H_n$

where k_1, k_2, k_3, k_4 +ve constants.

$$O_{n+1} = (1+k_1)O_n - k_3 O_n H_n \quad \text{and} \quad H_{n+1} = (1+k_2)H_n - k_4 O_n H_n$$

Using data collected by birders, and "fitting" the data to this model, we find / are told: $\underbrace{k_1 = 0.2,\ k_3 = 0.001}_{\text{owls}},\ \underbrace{k_2 = 0.3,\ k_4 = 0.002}_{\text{hawks}}$

$$O_{n+1} = (1.2)O_n - 0.001\, O_n H_n \quad \text{and} \quad H_{n+1} = (1.3)H_n - 0.002\, O_n H_n$$

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Equillibrium values?

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Equillibrium values?

If there are eq. values O & H then $O = (1.2)O - (0.001)OH$
& $H = (1.3)H - (0.002)OH$

That is $(O, H) = (150, 200)$ is the eq. value.

What does this mean?

$$O_{n+1} = (1+k_1)O_n - k_3 O_n H_n \quad \text{and} \quad H_{n+1} = (1+k_2)H_n - k_4 O_n H_n$$

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What does this mean? If initial populations are $O_0 = 150$ & $H_0 = 200$ then they will stay the same.

$$O_{n+1} = (1+k_1)O_n - k_3 O_n H_n \quad \text{and} \quad H_{n+1} = (1+k_2)H_n - k_4 O_n H_n$$

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That is $(O, H) = (150, 200)$ is the eq. value.

What does this mean? If initial populations are $O_0 = 150$ & $H_0 = 200$ then they will stay the same.

What if the two populations are not exactly 150 & 200? Will both populations survive? Will one dominate?

We investigate experimentally the change in each population starting from different initial populations:

Case 1 $O_0 = 151$ and $H_0 = 199$

Case 2 $O_0 = 149$ and $H_0 = 201$

We investigate experimentally the change in each population starting from different initial populations:

Case 1 $O_0 = 151$ and $H_0 = 199$

n	O_n	H_n
0	151	199
1	⋮	⋮
2		
⋮		

Table of predicted pop. & plot

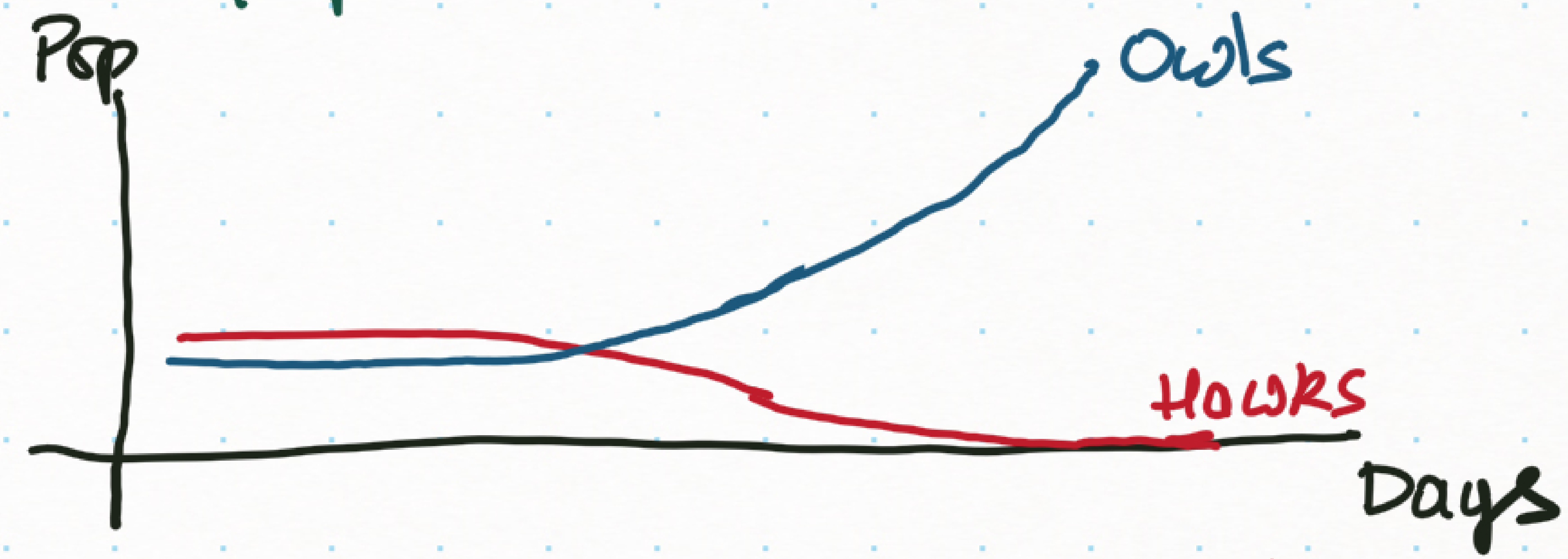
Case 2 $O_0 = 149$ and $H_0 = 201$

n	O_n	H_n
0	149	201
1	⋮	⋮
2		
⋮		

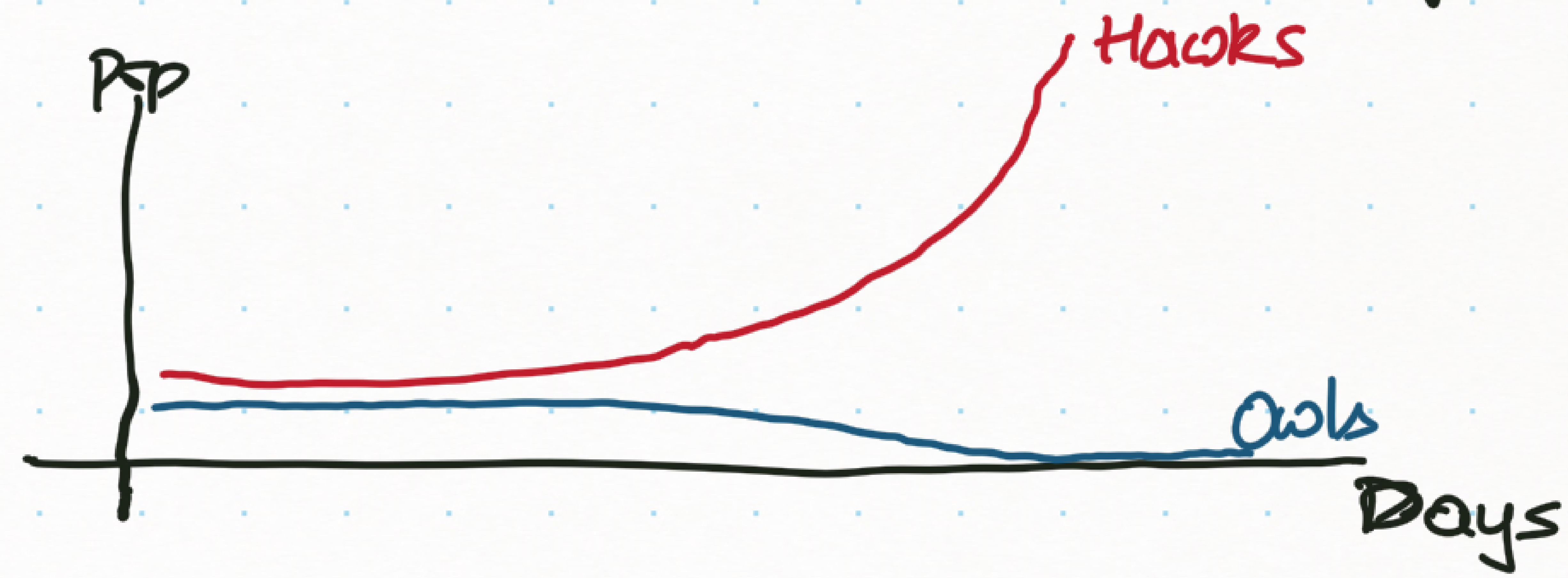
Table of predicted pop. & plot

We investigate experimentally the change in each population starting from different initial populations:

<u>Case 1</u> $O_0 = 151$ and $H_0 = 199$



<u>Case 2</u> $O_0 = 149$ and $H_0 = 201$



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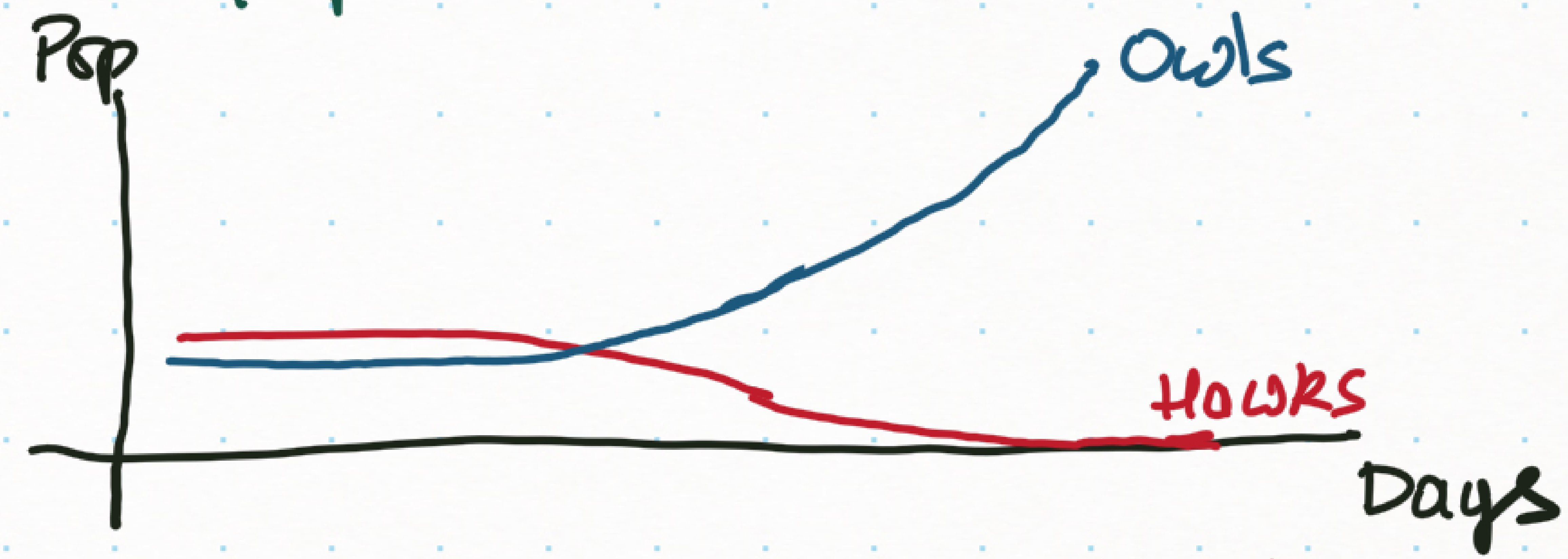

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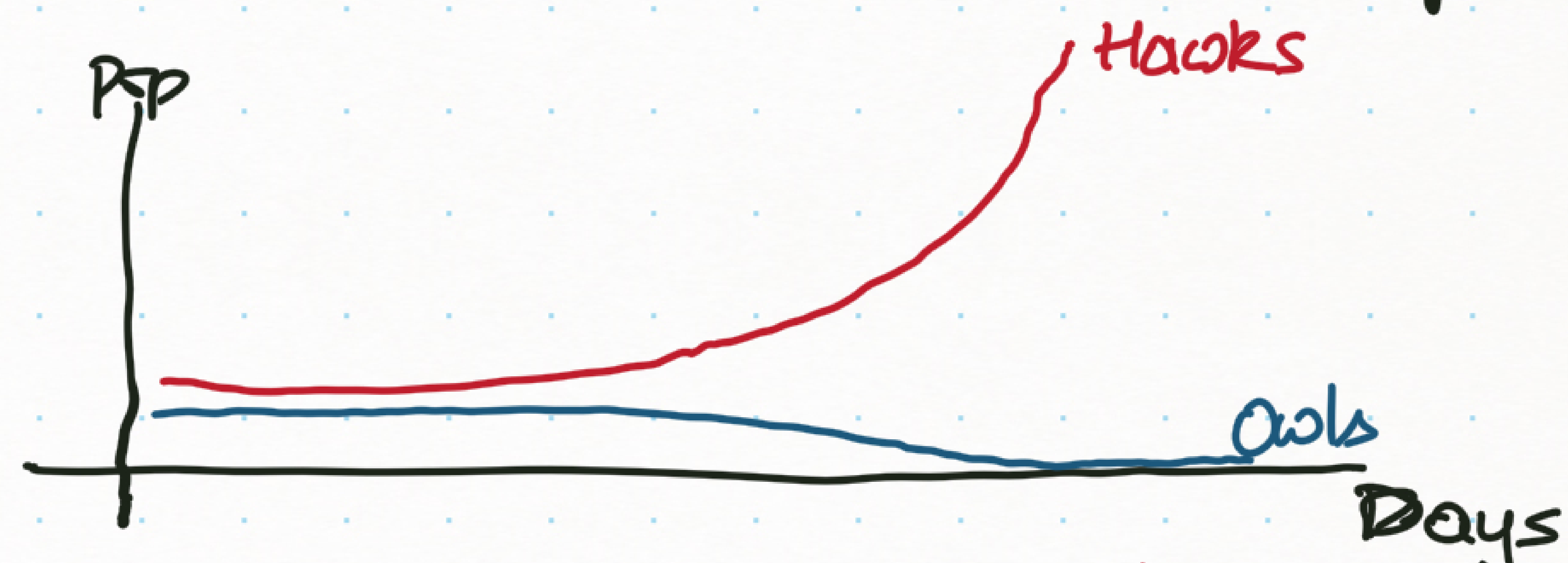

Case 3 $O_0 = 10$ and $H_0 = 10$