Math 454/553 — Hemanshu Kaul

Here is an example of how to write proofs in your HW solutions:

Ques Prove that every simple graph with at least two vertices has two vertices of equal degree. Is the conclusion true for graphs in general? What about loopless graphs?

(I was going to ask this in the 1st mid-term but ..... :-))

Soln

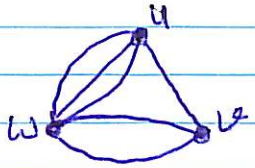
Proof: The degree of a vertex in an n-vertex simple graph is in the set $\{0, 1, 2, \ldots, n-1\}$. These are n distinct values.
If no two vertices have the same degree, then all these n values appear as a degree of some vertex.
But, a graph cannot have vertices with degree equal to 0 (not adjacent to any vertex) and degree equal to $n-1$ (adjacent to all other vertices).
Hence, these have to be two vertices with same degree. QED

This does not hold for graphs with loops:

e.g. 

$\deg(v) = 1$ & $\deg(u) = 3$ (loop contributes 2 to the degree)

This does not hold for general loopless graphs:

e.g. 

$\deg(u) = 4$ & $\deg(w) = 5$, all distinct.
$\deg(v) = 3$

Comments on the HW problems:

#3 Describe what your graph $(K_{m,n})$ is like, by labeling its vertices. (Say explicitly what the labels are). The order the vertices in an appropriate way to get a nice adjacency matrix

#21 If a graph is bipartite, give a labeling of its vertices into two partite sets.
If a graph is not bipartite, how did we argue in class?

#22 If a pair of graphs is isomorphic, you can label their vertices and show an explicit isomorphism. This bijection can be proved to be an isomorphism using $A(G)$ as in Example 1.1.21. Also read & Example 1.1.30 for using $\overline{G}$ instead of G.
Sometimes redrawing a graph to mimic the other graph is enough to show an isomorphism.

To show two graphs are not isomorphic, you use the ideas in Example 1.1.30 or you show a graph which is a subgraph of one but not the other graph.
These comments also apply to #21.

#29 We discussed in class, how to write the proof using two cases based on the degree of a vertex in a 6-vertex graph G.

#31
#35 Both these problems have 'if and only if' statements so, make sure you prove both directions $\Rightarrow$ & $\Leftarrow$.

$\boxed{\Rightarrow}$ You have to prove the given graph has certain kind of number of vertices (n).

$\boxed{\Leftarrow}$ For the given kind of n, you have show the required graph / decomposition exists.