HANDOUT (Section 7.2)

① Illustration of proof of Theorem 7.2

Let $m=4$, $n=9$. Note $\phi(m)=2$, $\phi(n)=6$ & $\phi(mn)=12$

The 9X4 array of numbers $1, 2, \ldots, 36$

Since 1 & 3 are co-prime to 4, the entries in the two columns below 1 & 3 are all co-prime to 4. These are marked with □ boxes. This indicates that $n\cdot\phi(m)=9\cdot 2=18$ numbers are co-prime to 4.

$\boxed{1}$	2	$\boxed{3}$	4
$\boxed{5}$	6	$\boxed{7}$	8
$\boxed{9}$	10	$\boxed{11}$	12
$\boxed{13}$	14	$\boxed{15}$	16
$\boxed{17}$	18	$\boxed{19}$	20
$\boxed{21}$	22	$\boxed{23}$	24
$\boxed{25}$	26	$\boxed{27}$	28
$\boxed{29}$	30	$\boxed{31}$	32
$\boxed{33}$	34	$\boxed{35}$	36

Within the two columns of entries co-prime to 4, we find numbers that are co-prime to $n=9$ as well. These are marked with ○ circles. This gives a total of $\phi(n)\phi(m)=6\cdot 2=12$ numbers co-prime to both n & m.

② $\phi(5040)=\phi(2^4\cdot 3^2\cdot 5\cdot 7)$
$=\phi(2^4)\phi(3^2)\phi(5)\phi(7) = 8\cdot 6\cdot 4\cdot 6 = 1152$

③ If n is odd, then $\phi(2n)=\phi(2)\phi(n)$ (∵ 2 & n are co-prime)
$= 1\cdot\phi(n) = \phi(n)$

④ If n is even, say $n=2^k m$ with m odd, then $\phi(2n)=\phi(2^{k+1}m)=\phi(2^{k+1})\phi(m)$ (∵ 2^{k+1} & m are coprime)
$= 2^k\phi(m)$
$= 2(2^{k-1}\phi(m)) = 2(\phi(2^k m)) = 2\phi(n)$

⑤ If n & $n+2$ are twin primes, then
$\phi(n+2)=n+1=(n-1)+2=\phi(n)+2$

⑥ If p and $2p+1$ are both odd primes, then
$\phi(4p+2)=\phi(2(2p+1))=\phi(2)\phi(2p+1)$ (∵ 2 & $2p+1$ are coprime)
$= 1\cdot(2p) = 2(p-1)+2 = \phi(4)\phi(p)+2 = \phi(4p)+2$