Counterexample for a statement true for $n \leq 10^7$

We mentioned in class that many of famous open problems (conjectures) in number theory have been verified computationally for all n up to an extremely large integer. However, this only gives evidence towards believing these conjectures but do not constitute a proof.

The example below shows that such evidence can sometimes be misleading.

Defn $\mathcal{E}_e(n)$ = number of integers of even-type $\leq n$
$\mathcal{O}(n)$ = number of integers of odd-type $\leq n$

where a positive integer is said to be of even-type (odd-type) if it has even (odd) number of primes in its prime factorization.

e.g. $6 = 2 \times 3$ is even-type, $8 = 2 \times 2 \times 2$ is odd-type
$45 = 3 \times 3 \times 5$ is odd-type.

Polya's Conjecture (1919) $\mathcal{E}_e(n) \leq \mathcal{O}(n)$ for $n \geq 2$

This was computationally verified for all $n \leq 10^7$ and many people believed it to be true.

However, in 1962 Lehman found a counterexample at $n = 906180359$.