

## Another Proof of " $\sqrt{2}$ is irrational"

### Fermat's method of infinite descent:

This is disguised form of getting a contradiction to the Well Ordering Principle by producing an infinite sequence of decreasing positive integers if  $\sqrt{2}$  is assumed to be rational.

$$\text{Suppose } \sqrt{2} = \frac{a}{b}$$

$$\text{Let } a_1 = 2b - a \text{ \& } b_1 = a - b$$

$$\text{then } \frac{a}{b} = \frac{a_1}{b_1} \quad (\because a^2 = 2b^2)$$

$$\text{Also, } 0 < b_1 < b$$

$$\text{since } 1 < \frac{a}{b} < 2 \Rightarrow b < a < 2b \Rightarrow 0 < a - b < b$$

$$\text{Now, define } a_{k+1} = 2b_k - a_k \text{ \& } b_{k+1} = a_k - b_k, \quad k \geq 1$$

$$\text{\& by same reasoning we get } \sqrt{2} = \frac{a}{b} = \frac{a_{k+1}}{b_{k+1}}$$

$$\text{and } 0 < b_{k+1} < b_k < b, \quad k \geq 1$$

Giving an infinite <sup>decreasing</sup> sequence of positive integers  $\langle b_k \rangle_{k=1}^{\infty}$  which is not possible.

Question: Use this idea to prove that  $\sqrt{N}$  is irrational where  $N$  is not a ~~perfect~~ square  
(How will you define  $a_k$  \&  $b_k$  in this case?)