Math 454/553 — Hemanshu Kaul

Here is an example of how to write proofs in your HW solutions:

<u>Ques</u> Prove that every simple graph with at least two vertices has two vertices of equal degree. Is the conclusion true for graphs in general? What about loopless graphs?

(I was going to ask this in the 1st mid-term but ...... :-))

<u>Soln</u>

<u>Proof</u>: The degree of a vertex in an n-vertex simple graph is in the set $\{0, 1, 2, \ldots, n-1\}$. These are n distinct values.
If no two vertices have the same degree, then all these n values appear as a degree of some vertex.
But, a graph cannot have vertices with degree equal to 0 (not adjacent to any vertex) and degree equal to $n-1$ (adjacent to all other vertices).
Hence, there have to be two vertices with same degree.
                                                    QED

This does not hold for graphs with loops:
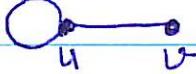
e.g. $\deg(v) = 1$ & $\deg(u) = 3$ (loop contributes 2 to the degree)

This does not hold for general loopless graphs:

e.g. $\deg(u) = 4$ & $\deg(w) = 5$, all distinct.
                                $\deg(v) = 3$