MATH 454/ 553 : Homework #1

Due Thursday, 9/4, in class before the lecture starts.

Re-read the " 'Why and How' of Homework" section of the course information sheet for some advice on the HWs for this course.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with it, don't hesitate to ask. You can contact me during office hours, or through email.

All problems require explicit and detailed proofs/ arguments/ reasons. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

You are allowed to discuss the homework problems with no one except your classmates, the TA, and the instructor. However, the solutions should be written by you and you alone in your own words. Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with.

If you discuss the problems with anyone, please note their name at the top of your HW submission under a subtitle "Collaborator" or "Discussed with:".

Problems 1 and 2 are compulsory for all students. Math 454 students submit a total of 4 problems while Math 553 students submit a total of 5 problems.

1. Let $\Delta(G)$ denote the maximum degree, and $\delta(G)$ denote the minimum degree of a vertex in a graph G.

Given a graph G and a subgraph H of G, consider the following two statements:

(1) $\delta(G) \geq \delta(H)$ and (2) $\Delta(G) \geq \Delta(H)$.

(a) Only one of these statements is true for all graphs. Which is it? Given a short proof of the truth of that statement.

(b) Find a counterexample showing the other statement is not always true.

2. Prove that every set of six people contains at least three mutual acquaintances or three mutual strangers. As the first step, first show how to convert this statement into a graph-theoretic statement, and then prove that graph-theoretic statement.

3. Let $N(n, k)$ denote the number of nonisomorphic simple graphs with n vertices and k edges. Compute $N(4, 3)$ by explicitly finding all such graphs.

Find all nonisomorphic simple graphs of order 4, by finding all $N(4, k)$ as above.

Comment: Remember that you have justify your claims and explain how you have considered all possibilities for graphs on 4 vertices.

4. Textbook exercise 1.1.30.

Comment: This simply uses the definition of matrix multiplication but be careful in writing your proof

- don't mix up your rows and columns.

5. Textbook exercise 1.1.31.
Comment: Note that this is an 'if and only if' statement which means you have to prove two implications - forward and backward. The forward implication asks you to prove that a self-complementary graph can only have a certain number of vertices. The backward implication is asking you to give a construction of a self-complementary graph on n vertices when n satisfies the divisibility condition. You have describe (use figures if you like) how such a graph on any number of vertices (not just specific small values of n) can be constructed.

6. Textbook exercise 1.1.22. Also, which of these graphs is bipartite?
Comment: If a graph is bipartite then simply give a labeling of its vertices using two labels that shows the bipartition. In addition to ideas discussed in class, read example 1.1.30 for another idea for showing (non)isomorphism