

MATH 454/ 553 : Homework #3

Due Thursday, 9/18, in class before the lecture starts.

Re-read the “ ‘Why and How’ of Homework” section of the course information sheet for some advice on the HWs for this course.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with it, don't hesitate to ask. You can contact me during office hours, or through email.

All problems require explicit and detailed proofs/ arguments/ reasons. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

You are allowed to discuss the homework problems with no one except your classmates, the TA, and the instructor. However, the solutions should be written by you and you alone in your own words. Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with.

If you discuss the problems with anyone, you are required to note their name at the top of your HW submission under a subtitle “Collaborator:” or “Discussed with:”.

Problems 1 and 2 are compulsory for all students. Math 454 students submit a total of 4 problems while Math 553 students submit a total of 5 problems.

1. Prove or disprove each of the following statements:

- (a) There exist infinitely many Eulerian graphs of even order and odd size.
- (b) Every Eulerian bipartite graph has an even number of edges.
- (c) Every maximal trail in an even graph is an Eulerian trail.

2. Solve each of the following (unrelated) problems:

- (a) Suppose in a bipartite graph all the vertices except one have the same degree d , and the remaining vertex has degree x . Show that x must be a multiple of d .
- (b) Define a simple graph R_k on vertex set of Q_k by making two binary k -tuples adjacent iff they agree in exactly one coordinate. Prove that R_k is isomorphic to Q_k if and only if k is even.

3. Textbook exercises 1.3.30 and 1.3.45.

4. Textbook exercise 1.2.39.

5. Textbook exercise 1.3.40.

6. Textbook exercise 1.3.44.