MATH 454/ 553 : Homework #4

Due Thursday, 9/25, in class before the lecture starts.

Re-read the " 'Why and How' of Homework" section of the course information sheet for some advice on the HWs for this course.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with it, don't hesitate to ask. You can contact me during office hours, or through email.

All problems require explicit and detailed proofs/ arguments/ reasons. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

You are allowed to discuss the homework problems with no one except your classmates, the TA, and the instructor. However, the solutions should be written by you and you alone in your own words. Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with.

NOTE - If you discuss the problems with anyone, you are required to note their name at the top of your HW submission under a subtitle "Collaborator:" or "Discussed with:".

Problems 1 and 2 are compulsory for all students. Math 454 students submit a total of 4 problems while Math 553 students submit a total of 5 problems.

1. Solve each of the following problems
(a) Consider the sequence $(5,5,5,5,4,4,4)$. Give two distinct arguments, including one by Havel-Hakimi theorem, to show whether or not this sequence is graphic.
(b) For which integers x $(0 \leq x \leq 7)$, if any, does $7,6,5,4,3,2,1,x$ give a graphic sequence? (Minimize the use of Havel-Hakimi in your proof.)
(c) Let G be a self-complementary graph of odd order. Prove that G contains a vertex of degree $(n(G)-1)/2$.

2. Prove or Disprove:
(a) When the algorithm for finding a bipartite subgraph from Section 1.3 is applied to a bipartite graph, it will always find the full graph as the solution.
(b) Every graph G has an orientation such that the out-degree and the in-degree of each vertex differs by at most 1.

3. Solve one of Textbook exercises 1.4.19 or 1.4.20.

4. Textbook exercise 1.3.53.

5. Textbook exercise 1.4.25.

6. Textbook exercise 1.4.36.