MATH 454/ 553 : Homework #7

Due Thursday, 10/23, in class before the lecture starts.

Re-read the " 'Why and How' of Homework" section of the course information sheet for some advice on the HWs for this course.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with it, don't hesitate to ask. You can contact me during office hours, or through email.

All problems require explicit and detailed proofs/ arguments/ reasons. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

You are allowed to discuss the homework problems with no one except your classmates, the TA, and the instructor. However, the solutions should be written by you and you alone in your own words. Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with.

NOTE - If you discuss the problems with anyone, you are required to note their name at the top of your HW submission under a subtitle "Collaborator:" or "Discussed with:".

Problem 1 and one of (Problems 2 or 3) are compulsory for all students. Math 454 students submit a total of 4 problems while Math 553 students submit a total of 5 problems.

1. (a) Which of the following relations is true for every G and H, where H is a subgraph of G? Assume both G and H have minimum degree at least 1.
(i) $\alpha(H) \leq \alpha(G)$; (ii) $\alpha'(H) \leq \alpha'(G)$; (iii) $\beta(H) \leq \beta(G)$; (iv) $\beta'(H) \leq \beta'(G)$.

(b) Let G be a graph with n vertices and m edges.
Show that $\alpha(G) \leq m/\delta(G)$ and $\alpha(G) \geq n/(\Delta(G)+1)$.

2. For each $k \geq 2$, construct a k-regular graph with no perfect matching.

3. Consider the graph in Textbook exercise 3.1.28. Give two different proofs of nonexistence of a perfect matching in it - one using vertex covers, and another using Hall's theorem.

4. Textbook exercise 3.1.29.

5. Textbook exercise 3.1.39.

6. Textbook exercise 3.1.18.