

MATH 454/ 553 : Homework #9

Due Thursday, 11/13, in class before the lecture starts.

Re-read the “‘Why and How’ of Homework” section of the course information sheet for some advice on the HWs for this course.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with it, don't hesitate to ask. You can contact me during office hours, or through email.

All problems require explicit and detailed proofs/ arguments/ reasons. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

You are allowed to discuss the homework problems with no one except your classmates, the TA, and the instructor. However, the solutions should be written by you and you alone in your own words. Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with.

NOTE - If you discuss the problems with anyone, you are required to note their name at the top of your HW submission under a subtitle “Collaborator:” or “Discussed with:”.

Problem 1 and Problems 2 are compulsory for all students. Math 454 students submit a total of 4 problems while Math 553 students submit a total of 5 problems.

1. True or False? Give a short proof or a counterexample as appropriate.

(a) Let G be a 2-connected graph with distinct vertices u and v . Given a u, v -path P , there exists another u, v -path Q that is internally disjoint from P .

(b) Let G be a connected graph with at least 3 vertices. Form a new graph G' from G by putting a new edge between every pair of vertices with distance 2 in the graph G . Then, G' is 2-connected.

2. Use the Global Menger Theorem (Theorem 4.2.21: $\kappa(G) = \min\{\lambda(x, y) \mid x, y \in V(G)\}$) to prove the following.

(a) Let G be a k -connected graph with at least $2k$ vertices and let A and B be disjoint sets of k vertices each. Prove that there exist k paths between A and B that are pairwise completely disjoint.

(b) Let G be a k -connected graph with at least $2k$ vertices and let A and B disjoint sets of vertices. Let $u(a)$ for each $a \in A$, and $w(b)$ for each $b \in B$ be non-negative integers such that $\sum_{a \in A} u(a) = \sum_{b \in B} w(b) = k$. Prove that G has k pairwise internally disjoint paths between A and B so that $u(a)$ of them start at a and $w(b)$ of them start at b , for each $a \in A$ and $b \in B$.

3. Textbook exercise 4.1.8 (only for the graph on left).

4. Textbook exercise 4.1.15.

5. Textbook exercise 4.2.18.

6. Textbook exercise 4.2.23.