MATH 553 : Homework #4

Due Tuesday, 10/23, in class before the lecture starts.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with it, don't hesitate to ask. You can contact me during office hours, or through email.

All problems require explicit and detailed proofs/ arguments/ reasons. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

You are allowed to discuss the homework problems with no one except your classmates, and the instructor. However, the solutions should be written by you and you alone in your own words. Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with.

If you discuss the problems with anyone, please note their name at the top of your HW submission under a subtitle "Collaborator" or "Discussed with:".

Solve all three problems below.

1. Give short proofs for the following two (unrelated) statements:

(a) Let G be an n-vertex simple graph. If $\delta(G) \geq (n+k-2)/2$ then G is k-connected (where $1 \leq k \leq n-1$).

(b) Let $V(G) = \{v_1, v_2, \ldots, v_n\}$ (with $n \geq 3$) and for each i, let G_i be the subgraph obtained by deleting the vertex v_i from G. Then G is connected if and only if at least two of these subgraphs are connected.

2. True or False? Give a short proof or a counterexample as appropriate.

(a) Let G be a 2-connected graph with distinct vertices u and v. Given a u,v-path P, there exists another u,v-path Q that is internally disjoint from P.

(b) Let G be a connected graph with at least 3 vertices. Form a new graph G' from G by putting a new edge between every pair of vertices at distance 2 in the graph G. Then, G' is 2-connected.

3. Use Menger's Theorem to prove the following.

(a) Let G be a k-connected graph with at least $2k$ vertices and let A and B be disjoint sets of k vertices each. Prove that there exist k paths between A and B that are pairwise completely disjoint. [Hint: Use Expansion Lemma to modify the given graph before applying Menger's theorem.]

(b) Prove König-Egerváry Theorem ($\alpha'(G) = \beta(G)$ when G is bipartite).