What is this course *really* about?
aka My aim for this course

Mathematicians of the eighteenth century, as present-day high school students, held Calculus as the pinnacle of mathematical sophistication. It was an exciting time in the development of calculus, which had reached a level of maturity after more than 2000 years of gradual development all over the world. There was this sudden explosion in ideas, methods, and applications of Calculus to Physics, Astronomy, and Engineering. Just as we do in present-day Calculus courses, nobody was worried about 'when and why' of these methods, and only concern was somehow applying them. But, by the turn of nineteenth century, mathematicians had to finally reckon with the fundamental limitations of these calculus based methods, starting from fundamental definitions of basic concepts like numbers, limits, convergence, etc. We will study these rigorous foundations of calculus in this course.

According to Underwood Dudley, there are at least eight levels of mathematical understanding:

1. Being able to do arithmetic
2. Being able to substitute numbers in 'formulas'/ being able to state or use elementary properties of concepts
3. Given 'formulas'/ elementary properties of a concept, being able to get other 'formulas'/ elementary properties
4. Being able to understand hypotheses and conclusions of theorems
5. Being able to understand the proofs of theorems, step by step
6. Being able to *really* understand proofs of theorems: that is, seeing why the proof is as it is, and comprehending the underlying ideas of the proof and its relation to other proofs and theorems
7. Being able to generalize and extend theorems, and apply them to seemingly unrelated problems
8. Being able to see new relationships, and discover and prove entirely new theorems.

The word 'theorem' is used above in a very general sense - it can also represent algorithms and techniques with a mathematical basis.

Levels 5 and 6 would be considered basic mathematical ability for Math majors. Non-trivial applications of Mathematics would lie in-between levels 6 and 7. While levels 7 and 8 constitute research in Mathematics. A lot of computer science, physics, and engineering, is deep applied mathematics and requires understanding at or beyond levels 6 and 7.

Calculus courses focus on a mixture of 1 and 2. While beginning proof based courses like Math 230 (Introduction to Discrete Mathematics) and Math 332 (Elementary Linear Algebra) focus on 3 and 4 with a bit of 5.

This course (Math 400) uses calculus as the starting point of examples and methods, and focuses on the upper part of levels 4, 5, and 6. The aim is give you a rigorous foundation in single-variable calculus - real number system; limits; convergence of sequences and series; continuity, differentiability, and integrability of functions - in levels up to 6, so that you can go onto levels 7 and 8, both as mathematicians and scientists/engineers, through the study of mathematical analysis, the repository of mathematical ideas underlying all of modern science and engineering.

I hope this course will help you make progress through these levels of mathematical understanding, and mathematical maturity. I would consider this a successful course if you gain confidence in your ability to read, understand, and write mathematical arguments (including proofs), especially as compared to the beginning of the semester. And, you feel that you can independently read, understand, and apply any concept or method from Real Analysis that you might need later on in your career.

with best wishes,
Hemanshu Kaul