Math 400: Discussion Questions # 1[1]

A statement listed with [T/F] is a True/False statement that requires a proof or a counterexample, as appropriate.

1. How does the proof that $\sqrt{2}$ is irrational change if we consider $\sqrt{3}$? $\sqrt{4}$?
2. Why is neither $\mathbb{N}$ nor $\mathbb{Z}$ a field?
3. The sets $\mathbb{N}$ and $\mathbb{Z}$ have a natural order. So does $\mathbb{Q}$. How about $\mathbb{R}$? How about $\mathbb{Z}_{12}$, the numbers $\{0,1,2,\ldots, 11\}$ under addition modulo 12?
4. How can we construct $\mathbb{R}$ from $\mathbb{Q}$?
5. Which of these sets contain $\sqrt{2}$?
   (a) $A = \{x \in \mathbb{Q} : x^2 < 3\}$
   (b) $B = \{x \in \mathbb{R} : x^2 < 3\}$
   (c) $C = A \cup B$
   (d) $D = A \cap B$
   (e) $E = A^c$

6. [T/F] For every pair of real numbers a and b, $|a + b| \leq |a| + |b|$.
7. [T/F] For every pair of real numbers a and b, $|a + b| \leq |a| - |b|$.
8. [T/F] For every pair of real numbers a and b, $|a - b| \leq |a| + |b|$.
9. [T/F] For every pair of real numbers a and b, $|a - b| \leq |a| - |b|$.

10. [T/F] Given $a, b \in \mathbb{R}$, we have $a = b$ if for some $\epsilon > 0$, it follows $|a - b| < \epsilon$.
11. [T/F] Given $a, b \in \mathbb{R}$, we have $a = b$ only if for some $\epsilon > 0$, it follows $|a - b| < \epsilon$.
12. [T/F] Given $a, b \in \mathbb{R}$, we have $a = b$ if for every $\epsilon > 0$, it follows $|a - b| < \epsilon$.
13. [T/F] Given $a, b \in \mathbb{R}$, we have $a = b$ only if for every $\epsilon > 0$, it follows $|a - b| < \epsilon$.

14. Let $A = \{\frac{n}{n+1} : n \in \mathbb{N}\}$. Which, if any, of these numbers is an upper bound for A: $\frac{1}{2}, 1, 6$?
15. What is the relation between maximum and supremum of a set? When are they not equal?
16. [T/F] An upper bound for a set $A \subset \mathbb{R}$ is necessarily an element of A.
17. [T/F] A least upper bound for a set $A \subset \mathbb{R}$ is necessarily an element of A.

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18. [T/F] A set $A \subset \mathbb{R}$ has at least one maximum. What if $A \subset \mathbb{N}$? $A \subset \mathbb{Q}$?
19. [T/F] A set $A \subset \mathbb{R}$ has at most one maximum.
20. [T/F] A set $A \subset \mathbb{R}$ has at least one upper bound.
21. [T/F] A set $A \subset \mathbb{R}$ has at most one upper bound.
22. [T/F] A set $A \subset \mathbb{R}$ has at least one least upper bound.
23. [T/F] A set $A \subset \mathbb{R}$ has at most one least upper bound.