Math 400: Discussion Questions # 10

A statement listed with [T/F] is a True/False statement that requires a proof or a counterexample, as appropriate.

1. How is the Mean Value Theorem reduced to Rolle's theorem? Explain and show the details.
2. Let f and g be differentiable functions on an interval I. Suppose $f' = g'$ on I. Show that $f(x) = g(x) + k$ for a fixed constant k.
3. Show that $e^x = 1 - x$ has only one solution in $\mathbb{R}$.
4. Use MVT to show that $\ln(1 + x) \leq x$ for any $x \geq 0$.
5. Use MVT to show that $\sin x < x$ for any $x \in (0, \frac{\pi}{2}]$.
6. Evaluate $\lim\limits_{x\to 0} \frac{1-\cos x}{x \sin x}$.
7. [T/F] Let $f_n(x) = \frac{x^2+nx}{n}$. (f_n) converges pointwise to $f(x) = x$.
8. [T/F] Let $f_n(x) = \frac{x^2+nx}{n}$. (f_n) converges uniformly to $f(x) = x$ on $\mathbb{R}$.
9. [T/F] Let $f_n(x) = \frac{x^2+nx}{n}$. (f_n) converges uniformly to $f(x) = x$ on $[-2, 2]$.
10. [T/F] Let $g_n(x)$ and $g(x)$ be differentiable functions on an compact interval I. Suppose g_n converges uniformly to g on I. Then, $\lim g_n'(x) = g'(x)$ on I.
11. [T/F] Let $g_n(x)$ and $g(x)$ be differentiable functions on an compact interval I. Suppose g_n converges pointwise to g on I, and g_n' converges uniformly to a function f on I. Then, $g'(x) = f(x)$ on I.
12. [T/F] $f_n(x) = \frac{1}{1+(nx-1)^2}$ converges uniformly on $[0, 1]$.
13. [T/F] $\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$ is uniformly convergent on $\mathbb{R}$.
14. [T/F] $\sum_{n=1}^{\infty} \frac{\sin nx}{n!}$ is uniformly convergent on $\mathbb{R}$.
15. [T/F] $\sum_{n=1}^{\infty} x^n \sin nx$ is uniformly convergent on $[-\frac{1}{2}, \frac{1}{2}]$.
16. [T/F] $\sum_{n=1}^{\infty} \frac{\cos nx}{n\sqrt{n+1}}$ is a continuous function on $\mathbb{R}$.
17. Find an interval such that $\sum_{n=1}^{\infty} \frac{x^{2n}}{n 2^n}$ is a continuous function on that interval.