Math 400: Discussion Questions # 3

1. Complete the proof of *uniqueness of limit* as outlined in the lecture.
2. Show that the sequence $a_n = (\frac{n+1}{n})$ converges to 1.
   (a) What value of N should we use?
   (b) Fill in the rest of the details of the proof.
3. Show that the sequence $b_n = (5 - \frac{1}{n^2})$ converges to 5.
   (a) What value of N should we use?
   (b) Fill in the rest of the details of the proof.
4. Consider the sequence $c_n = \frac{\sin(n^2)}{n^2}$.
   (a) Evaluate the initial terms of this sequence. Are they getting closer to a particular value?
   (b) Is there a limit of this sequence?
5. Write and explain the negation of the definition of convergence of sequence.
6. What is the long-term behavior of the sequence $d_n = (1 - n^2)$?
7. [T/F] Every convergent sequence is bounded.
8. [T/F] Every bounded sequence is convergent.
9. Let $(a_n) \to a$.
   (a) [T/F] There exists N s.t. $a - 1 < a_n < a + 1$ for all $n \geq N$.
   (b) [T/F] $L \leq a_n \leq U$ for all n,
   where $L = \min\{a_1, a_2, \ldots a_{N-1}, a - 1\}$, and $U = \max\{a_1, a_2, \ldots a_{N-1}, a + 1\}$.
10. [T/F] If $(a_n + b_n) \to a + b$, then $(a_n) \to a$ and $(b_n) \to b$.
11. [T/F] If $(a_n) \to a$ and $a_n \geq 0$ for all n, then $a \geq 0$.
12. [T/F] If $(a_n) \to a$ and $a_n \geq 0$ for all $n \geq N$, then $a \geq 0$.
13. [T/F] If $(a_n) \to a$ and $(b_n) \to b$, with $a_n \geq b_n$ for all n, then $a \geq b$.
14. [T/F] If $(a_n) \to a$ and $(b_n) \to b$, with $a_n \geq b_n$ for all $n \geq N$, then $a \geq b$.