Math 400: Discussion Questions # 5

A statement listed with [T/F] is a True/False statement that requires a proof or a counterexample, as appropriate.

1. [T/F] If, for every $\epsilon > 0$ there exists N s.t. $|a_{n+1} - a_n| < \epsilon$ for all $n \geq N$, then (a_n) is a Cauchy sequence.
2. [T/F] If a sequence (a_n) with each term in $\mathbb{N}$ converges to an element of $\mathbb{N}$, then (a_n) is a Cauchy sequence.
3. [T/F] If a sequence (a_n) with each term in $\mathbb{N}$ is a Cauchy sequence, then (a_n) converges to an element of $\mathbb{N}$.
4. [T/F] If a sequence (a_n) with each term in $\mathbb{Q}$ converges to an element of $\mathbb{Q}$, then (a_n) is a Cauchy sequence.
5. [T/F] **If a sequence (a_n) with each term in $\mathbb{Q}$ is a Cauchy sequence, then (a_n) converges to an element of $\mathbb{Q}$.**
6. [T/F] Every Cauchy sequence is bounded.
7. [T/F] Every bounded sequence is Cauchy.
8. [T/F] Every Cauchy sequence is monotone.
9. [T/F] Every monotone sequence is Cauchy.
10. Complete the proof of "(a_n) convergent implies (a_n) Cauchy".
11. [T/F] A sequence is Cauchy (in $\mathbb{R}$) if and only if it is convergent (in $\mathbb{R}$).

12. [T/F] If a sequence (a_n) is Cauchy, then the series $\sum a_n$ converges.
13. [T/F] If a series $\sum a_n$ converges, then the sequence (a_n) is Cauchy.
14. [T/F] There exists a convergent series $\sum a_n$ with $\lim a_n \neq 0$.
15. Give reasons for each of the four steps in the proof of "Algebra of Series Limits"
16. [T/F] If a series $\sum a_n$ converges, then the series $\sum |a_n|$ converges.
17. [T/F] If a series $\sum |a_n|$ converges, then the series $\sum a_n$ converges.
18. [T/F] There exists a series that is absolutely convergent but not convergent.
19. [T/F] There exists a series that is convergent but not absolutely convergent.
20. [T/F] There exists a series that are both convergent and absolutely convergent.
21. [T/F] If $\sum a_n = A$ and $\sum b_n = B$, then $\sum a_n b_n = AB$.