

## Math 400: Discussion Questions # 6

A statement listed with [T/F] is a True/False statement that requires a proof or a counterexample, as appropriate.

1. Review for Cantor Set:
  - (a) How many intervals are in the stage 5 of the Cantor set? What is the length of each of those intervals?
  - (b) What is the length of the Cantor set? Justification?
  - (c) What is the cardinality of the Cantor set? Justification?
  - (d) What is the dimension of the Cantor set? Justification?
  - (e) What is the definition of Sierpinski set? the definition of Menger cube?
  
2. Is the empty set open? Is  $\mathbb{R}$  open?
3. Is  $\mathbb{R} \setminus \{5\}$  open?
4. Is  $\{5\}$  open?
5. Is  $\mathbb{Q}$  open?
6. Is the interval  $(2, 5)$  open?
7. Is the interval  $[2, 5]$  open?
8. Is the interval  $(2, 5]$  open?
9. Is the interval  $(a, \infty)$  open?
10. Is the interval  $(-\infty, b)$  open?
11. [T/F] Union of countably many open sets is an open set.
12. [T/F] Union of uncountably many open sets is an open set.
13. [T/F] Intersection of finitely many open sets is an open set.
14. [T/F] Intersection of countably many open sets is an open set.
  
15. Is the empty set closed? Is  $\mathbb{R}$  closed?
16. Is  $\mathbb{R} \setminus \{5\}$  closed?
17. Is  $\{5\}$  closed?
18. Is  $\mathbb{Q}$  closed?
19. Is the interval  $(2, 5)$  closed?
20. Is the interval  $[2, 5]$  closed?
21. Is the interval  $(2, 5]$  closed?

22. Is the interval  $(a, \infty)$  closed?
23. Is the interval  $(-\infty, b)$  closed?
24. Give an example of a set that both open and closed.
25. Give an example of a set that neither open nor closed.
26. [T/F] Union of finitely many closed sets is a closed set.
27. [T/F] Union of countably many closed sets is a closed set.
28. [T/F] Intersection of countably many closed sets is a closed set.
29. [T/F] Intersection of uncountably many closed sets is a closed set.
30. [T/F] If  $a$  is a limit point of  $A$ , then  $a \in A$ .
31. [T/F]  $a$  is a limit point of  $A$  iff there is a sequence  $(a_n) \subset A$  with  $a_n \rightarrow a$ .
32. [T/F] If a set is closed then it contains all its limit points.
33. [T/F] If a set contains all its limit points then its closed.
34. [T/F] There exists an open set that contains all its limit points.
35. [T/F] IF a set contains all its limit points then its not open.
36. What is the closure of  $\{5\}$ ?
37. What is the closure of  $\{5, 6\}$ ?
38. What is the closure of  $(2, 4)$ ?
39. What is the closure of  $(3, 5]$ ?
40. What is the closure of  $\mathbb{Q}$ ?
41. What is the closure of  $\mathbb{R}$ ?
42. [T/F]  $A \subseteq \bar{A}$ .
43. [T/F]  $\bar{\bar{A}} \subseteq A$ .
44. [T/F]  $\text{Closure}(\bar{A}) = \bar{A}$ .