

## Math 400: Discussion Questions # 7

A statement listed with [T/F] is a True/False statement that requires a proof or a counterexample, as appropriate.

1. Is  $\{3\}$  compact?
2. Is the Cantor set compact?
3. Is  $\{x \in \mathbb{R} : 2 < |x| < 4\}$  compact?
4. Is  $\{x \in \mathbb{R} : 2 \leq |x| \leq 4\}$  compact?
5. Is  $\{-1, 2, \pi, 7, -16\}$  compact? Give two different proofs for your answer.
6. Is  $\mathbb{R}$  compact? Give two different proofs for your answer.
7. Consider the set  $(0, 4)$  and the collection  $\mathcal{U} = \{(\frac{1}{k}, 4 - \frac{1}{k})\}_{k=1}^{\infty}$ .
  - (a) Verify that  $\mathcal{U}$  is an open cover of  $(0, 4)$ .
  - (b) Verify that  $\mathcal{U}$  does not have a finite subcover of  $(0, 4)$ .
  - (c) What does this tell about  $(0, 4)$ ?
8. [T/F] If  $f(x) = 4x + 8$ , then  $\lim_{x \rightarrow 3} f(x) = 20$ .
9. Complete the proof of  $\lim_{x \rightarrow 2} x^2 = 4$ .
10. [T/F] If  $f(x) = 3x - 2$ , then  $\lim_{x \rightarrow 4} f(x) = 20$ .
11. [T/F]  $\lim_{x \rightarrow 0} \sin(\frac{1}{x}) = 0$
12. [T/F]  $\lim_{x \rightarrow 0} \sin(\frac{1}{x}) = 1$