Math 400: Discussion Questions # 9

A statement listed with [T/F] is a True/False statement that requires a proof or a counterexample, as appropriate.

1. [T/F] Fix $a, b \in \mathbb{R}$. $f(x) = ax + b$ is uniformly continuous on $\mathbb{R}$.
2. [T/F] Fix $a \in \mathbb{R}$. $f(x) = x^2$ is uniformly continuous on $[0, a]$.
3. [T/F] $f(x) = x^2$ is uniformly continuous on $\mathbb{R}^+$.
4. [T/F] $f(x) = \sin x$ is uniformly continuous on $\mathbb{R}$.
5. [T/F] $f(x) = \cos \frac{1}{x}$ is uniformly continuous on $(0, 1)$.
6. [T/F] $f(x) = x^{17} \sin x - e^x \cos 3x$ is uniformly continuous on $[0, \pi]$.
7. [T/F] $f(x) = x^2 \sin \frac{1}{x}$ is uniformly continuous on (0,1]
8. [T/F] $f(x) = \frac{1}{x}$ is uniformly continuous on $(\frac{1}{2}, \infty)$.
9. [T/F] If f is continuous at c then f is differentiable at c.
10. [T/F] If f is differentiable at c then f is continuous at c.
11. [T/F] If f is differentiable at c then f' is continuous at c.
12. [T/F] The following is an acceptable proof for the Chain rule:
    $(g \circ f)'(c) = \lim\limits_{x \to c} \frac{g(f(x)) - g(f(c))}{x - c} = \lim\limits_{x \to c} \frac{g(f(x)) - g(f(c))}{x - c} \frac{f(x) - f(c)}{f(x) - f(c)} = \lim\limits_{x \to c} \frac{g(f(x)) - g(f(c))}{f(x) - f(c)} \frac{f(x) - f(c)}{x - c} = \lim\limits_{x \to c} \frac{g(f(x)) - g(f(c))}{f(x) - f(c)} \lim\limits_{x \to c} \frac{f(x) - f(c)}{x - c} = g(f'(c))f'(c)$.
13. What is the derivative of $f(x) = |x|$ on $\mathbb{R}$?
14. Let $f(x) = x|x|$. Does $f'(0)$ exist?
15. Let $v(x)$ be differentiable at c and $v(c) \neq 0$. Derive a rule (directly) for the derivative of $1/v(x)$ at c.
16. Prove the Quotient rule for differentiation using the Product rule.
17. Let f be differentiable on (a, b). How can we find the max and min values of f on (a, b)? How can we find the max and min values of f on $[a, b]$?
18. Let f be differentiable, that is f' exists. Is f' differentiable? Is f' continuous? Does f' have the Intermediate Value Property?