MATH 400: Homework #3

Only the 'Submission Problems' listed below are due Wednesday, 9/16, before 11:59pm, via a PDF file uploaded to the Homework#3 under Assignments in the Blackboard course page.

You are allowed to discuss the homework problems with no one except your classmates, the TA, and the instructor. However, the solutions should be written by you and you alone in your own words. **If you discussed HW problems with a classmate or TA, you have to write their name at the top of the HW submission as a collaborator.** Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with.

Re-read the "Homework Assignment" and " 'Why and How' of Homework" sections of the course information sheet for some important advice on the HWs for this course.

All problems require explicit and detailed explanations. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with any HW problem, don't hesitate to ask me. You are encouraged to ask questions during the *Live Class on Blackboard*, through the *Blackboard Discussion Forums*, during the *Google Meet Office Hours*, during the *TA office hours*, or through *Email to me*.

PART I: Practice Problems

1. Pay close attention to and attempt at least the following exercises from the textbook.
   Section 2.2: #1, #2, #3, #4, #5, #7.
   Section 2.3: #1, #3 (Squeeze Theorem: make a note of this statement and use it in the future if needed), #4, #5, #7, #9, #11.

PART II: Submission Problems

2. **Submit written solutions to exercises as listed below:**

   Section 2.2: #2b, #5.
   Section 2.3: #1, #3, #7.

PART III: Readings, Comments, etc.

3. We discussed in class the idea that the initial portion of a sequence is irrelevant for understanding its long-term behavior. To make this idea even more precise, we can define what a tail of sequence is and then show that convergence of a sequence is the same as the convergence of its tail.

   Let $(a_n)_{n=1}^{\infty}$ be a sequence. For a fixed $K \in \mathbb{N}$, define the <u>*K-tail*</u> of the sequence as the sequence starting at the $(K+1)$ term, that is $(a_{K+1}, a_{K+2}, a_{K+3}, \ldots)$, or $(a_n)_{n=K+1}^{\infty}$, or $(a_{n+K})_{n=1}^{\infty}$. We are simply throwing away the first K terms from the sequence.

Show that the following three statements are equivalent (if one is true then all are true, and if one is false then all are false).:

(i) The sequence $(a_n)_{n=1}^{\infty}$ converges.

(ii) The K-tail sequence $(a_{n+K})_{n=1}^{\infty}$ converges for each $K \in \mathbb{N}$.

(iii) The K-tail sequence $(a_{n+K})_{n=1}^{\infty}$ converges for some $K \in \mathbb{N}$.

Furthermore, if any (and hence all) of the limits exist, then for any $K \in \mathbb{N}$ $\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+K}$.

4. Here is a seemingly obvious statement that requires some thought. Prove that the sequence $(n^{1/n})$ converges to 1. What is the value of N you would use?