MATH 400: Homework #6

Only the 'Submission Problems' listed below are due Wednesday, 10/14, before 11:59pm, via a PDF file uploaded to the Homework#6 under Assignments in the Blackboard course page.

You are allowed to discuss the homework problems with no one except your classmates, the TA, and the instructor. However, the solutions should be written by you and you alone in your own words. **If you discussed HW problems with a classmate or TA, you have to write their name at the top of the HW submission as a collaborator.** Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with.

Re-read the "Homework Assignment" and " 'Why and How' of Homework" sections of the course information sheet for some important advice on the HWs for this course.

All problems require explicit and detailed explanations. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with any HW problem, don't hesitate to ask me. You are encouraged to ask questions during the *Live Class on Blackboard*, through the *Blackboard Discussion Forums*, during the *Google Meet Office Hours*, during the *TA office hours*, or through *Email to me*.

PART I: Practice Problems

1. Pay close attention to and attempt at least the following exercises from the textbook. Section 3.2: #1, #2, #3, #5, #7, #8, #9, #11, #13, #14, #15.
2. Prove that the Cantor set is closed.

PART II: Submission Problems

3. **Submit written solutions to all 4 problems listed below.**
   - Section 3.2: #3, #9, #11, #13. (each worth 12.5 points)

PART III: Readings, Comments, etc.

4. Cantor Set $\mathcal{C}$ has many interesting properties. We proved/discussed some in class, try to prove the rest of them.
   - $\mathcal{C}$ is non-empty.
   - $\mathcal{C}$ has length zero.
   - $\mathcal{C}$ is a closed set.
   - $\mathcal{C}$ has no isolated points. That is, every point in $\mathcal{C}$ is a limit point of $\mathcal{C}$.

- $\mathcal{C}$ is uncountable.
- Let $x \in [0,1]$, then $x \in \mathcal{C}$ iff x can be expressed using only 0 and 2 in its ternary representation. (Ternary representation of a number means its Base 3 representation using only the numbers 0, 1, 2. For $x \in [0,1]$ this means $x = 0.a_1a_2a_3a_4\ldots a_n\ldots = \frac{a_1}{3} + \frac{a_2}{3^2} + \frac{a_3}{3^3} + \ldots + \frac{a_n}{3^n} + \ldots$. Here is the rule for choosing the first ternary digit a_1 is $a_1 = 0$ if $0 \leq x \leq \frac{1}{3}$, $a_1 = 1$ if $\frac{1}{3} < x \leq \frac{2}{3}$, $a_1 = 2$ if $\frac{2}{3} < x \leq 1$. And so on. Can you see the connection with $\mathcal{C}$ now?)
- Can you now give a proof for '$\mathcal{C}$ is uncountable' using the previous property?
- What if we modified the Cantor's construction for $\mathcal{C}$ by splitting each interval into four equal quarter intervals and then removing the second quarter interval, and so on. Would the set created by applying this rule to $[0,1]$ repeatedly give us something like $\mathcal{C}$? How would you describe this set using the language of k-nary representation?
- What if we modified the Cantor's construction for $\mathcal{C}$ by splitting each interval into two equal half intervals and then removing the second half interval, and so on. Would the set created by applying this rule to $[0,1]$ repeatedly give us something like $\mathcal{C}$? How would you describe this set using the language of k-nary representation?