MATH 400: Homework #7

Only the 'Submission Problems' listed below are due Wednesday, 10/21, before 11:59pm, via a PDF file uploaded to the Homework#7 under Assignments in the Blackboard course page.

You are allowed to discuss the homework problems with no one except your classmates, the TA, and the instructor. However, the solutions should be written by you and you alone in your own words. **If you discussed HW problems with a classmate or TA, you have to write their name at the top of the HW submission as a collaborator.** Any incident of plagiarism/ cheating (from a person or from any online resource) will be strictly dealt with.

Re-read the "Homework Assignment" and " 'Why and How' of Homework" sections of the course information sheet for some important advice on the HWs for this course.

All problems require explicit and detailed explanations. Solutions should be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with any HW problem, don't hesitate to ask me. You are encouraged to ask questions during the *Live Class on Blackboard*, through the *Blackboard Discussion Forums*, during the *Google Meet Office Hours*, during the *TA office hours*, or through *Email to me*.

PART I: Practice Problems

1. Pay close attention to and attempt at least the following exercises from the textbook.
   Section 3.3: #1 (note this important and useful fact; we will revisit this later in the course), #2, #3, #4, #5, #7, #11.
   Section 4.2: #1, #5, #6, #7, #9 (Note this definition for infinite limits), #11(Note this useful fact).

PART II: Submission Problems

2. **Submit written solutions to all 5 problems listed below.**
   - Section 3.3: #1, #2&11 (For any four parts out of the five, prove whether the given set is compact (using any equivalent definition) or not compact (by either showing sequential compactness doesn't hold or by showing the open cover definition doesn't hold)).
   - Section 4.2: #1ab, #5cd, #11 (Squeeze Theorem: Make a note of this).

PART III: Readings, Comments, etc.

3. Perfect sets (sets which are closed and contain no isolated points) are discussed in Section 3.4. Can you prove that Cantor set is perfect? See the argument in that Section. Also read

the proof of why every perfect sets is uncountable: its a beautiful application of Nested Compact Set Property.

4. Students interested in Topology and intending to take advanced courses in any type of Analysis are encouraged to read the portion of Section 3.4 on Connected sets. Also note that the intuitive notions of interior and boundary of a set can be made precise using the notion of ϵ-neighborhood that we are familiar with.

   $Int(A)$, interior of set A, is the set of points x such that there exists a neighborhood of x contained in A.

   $Ext(A)$, exterior of set A, is the set of points x such that there exists a neighborhood of x contained in A^c.

   $\partial(A)$, boundary of set A, is the set of points x such that every neighborhood of x contains points from both A and A^c.

   Check that these formal definitions match with your intuition of these concepts for various examples of sets A such as intervals, etc.

   Prove that for any set $A \subseteq \mathbb{R}$, $\mathbb{R} = Int(A) \cup \partial(A) \cup Ext(A)$, and each of these three sets is pairwise disjoint.

5. How did the concept of 'compactness' develop historically? Read the outline of its fascinating history in `https://arxiv.org/pdf/1006.4131.pdf`.

   How did what we call Real Analysis develop historically? This textbook follows the chronological order in discussion of the topics: `https://open.umn.edu/opentextbooks/textbooks/200`.