Math100 Project: why we use multilayer windows in winter

Introduction.
Winter in Chicago is really cold. Residents here use two-layer windows with a layer of air sandwiched in between two glass layers to achieve energy efficiency. In fact, this setup allows residents to reduce energy costs and lower their utility bills. Let's build a model to understand how it works and calculate how much you can save.

Assumptions.

- This is only heat conduction and no heat convection. That is the air between the two layers of glass does not flow.
- Indoor temperature is always T_{in} and outside temperature is always T_{out}. That is the heat conduction reaches steady state, or the heat flux is a constant.
- Glass is homogeneous and its thermal conduction coefficient k_g is a constant. The thickness of a single layer of glass (single layer) is d.
- The thickness of trapped air between the two glass is l, and its thermal conduction coefficient is k_a.

The model. Let's assume the heat flow Q from the high temperature T_1 to the outside low temperature T_2 follows $Q = k\dfrac{T_1 - T_2}{w}$, where w is the thickness of the medium and k is the thermal conduction coefficient.

Questions.

- Consider a window with only one layer glass of thickness $2d$. Calculate how much heat low from inside to outside, say Q_s.
- Consider a window with two layers of glass (the thickness is d for each layer) sandwiching a layer of air (thickness l). Calculate how much heat low from inside to outside, say Q_d.
- Calculate the ratio Q_d/Q_s.

- Let $k_g/k_a = 16$, plot Q_d/Q_s versus l/d. Comment what you get.
- What happens if you add one more layer air and one more layer glass? more layers?
- What can you learn from this problem?

Contact Prof. Shuwang Li at *sli15@iit.edu*
prerequisite: high school physics.