Math100 Project: A simple model for tumor growth

Introduction. It has been observed experimentally that a tumor grows by dividing its cells, and at early stage the tumor grows at a rate proportional to the volume of total dividing cells at that moment. Let $V(t)$ be the volume of total dividing cells at a time t, and the rate of change of V with respect to t reads

$$\frac{dV}{dt} = \lambda V, \tag{1}$$

where λ is some positive constant. Equation (1) is known as the law of natural growth. Given the initial tumor volume is V_0 at the initial time t_0, the solution to the above equation is $V(t) = V_0 e^{\lambda(t-t_0)}$. That is right, the dividing tumor cells grow exponentially with time. But when a tumor is detected, a patient usually goes through some medical procedure to control its growth, say for example by chemotherapy.

Question 1.

Suppose the tumor cells are killed at a constant emigration rate m by chemotherapy.

(a): Figure out the rate of change of the tumor cells at time t.

(b): Find the solution to the equation you got in (a), given the initial tumor volume is V_0 at the initial time $t_0 = 0$.

(c): What condition on m will lead to an exponential growth of the cells?

(d): What condition on m will lead to a constant tumor cells?

(e): What condition on m will lead to a decline of tumor cells?

Question 2.

Suppose the patient does not do any treatment, and let constant K be the carrying capacity, then Eq.(1) becomes:

$$\frac{dV}{dt} = \lambda V(1 - \frac{V(t)}{K}). \tag{2}$$

This equation is known as Verhulst model for logistic growth.

(a): Find the solution to the equation using the initial tumor volume is V_0 at the time $t_0 = 0$.

(b): What can you learn from this model?

Question 3.

In general, solid tumors do not grow exponentially fast in time. This is because as a tumor becomes

larger, the time for cell dividing continuously increases. Many researchers have found that the cell volume data for many solid tumors agree with an equation

$$V(t) = V_0 e^{[\frac{\lambda}{\beta}(1-e^{-\beta t})]}, \tag{3}$$

where β is another positive constant. Equation (3) is known as Gompertzian relation.
(a) Figure out the growth rate of tumor cells and explain how tumor volume grows under Gompertzian relation.
(b) Find the tumor volume as time $t \to \infty$.

For the above problems, please use Matlab to plot your solutions and do a parameter study. Then write a report summarizing your findings and what you learn, also please describe the broader applications of these models through a literature review.

Contact Prof. Shuwang Li at *sli15@iit.edu*
prerequisite: Basic Calculus. You should be able to solve the above equations at the end of Cal2 (Math 152).