

## MATH 100 : Project on ‘Symmetries of magic squares, and Tic Tac Toe’<sup>1</sup>

**Description:** Magic squares of order  $n$  are squares of cells filled with the numbers 1 through  $n^2$ , such that every row, column, and main and reverse diagonal have the same sum – the “magic sum.” They have been known to exist for at least 2600 years. This project is to investigate symmetries of magic squares, and produce new magic squares from old, and show that winning Tic Tac Toe corresponds to finding a row, column, diagonal, or reverse diagonal with the needed magic sum. Look up the Wikipedia article on Magic Squares.

You are expected to:

(i) Read and understand pages 1-7 of Chapter 10 (Magic Squares) and page 1 to the top of page 4 of Chapter 11 (Tic Tac Toe Magic) of Cleve Moler’s Experiments with MATLAB.

(ii) Work exercises #1, 2, and 6ab from Chapter 10 (Magic Squares).

(iii) Starting from the Lo Shu 3x3 magic square, generate all 3x3 magic squares by horizontal, vertical, diagonal, or counter-diagonal reflection, and by one or more 90-degree rotations. How many did you obtain?

(iv) Show that the “magic” property of a 3x3 or 4x4 magic square is preserved under these symmetries; i.e., after application the resulting square is also magic.

(v) Argue that the Lo Shu 3x3 magic square is the “only” 3x3 magic square as follows. First, argue why 5 must go in the middle. Then find where 9 must go. Proceed in this fashion until the positions of all numbers are completely determined. For all steps, if there is more than one cell a number could go in, identify the symmetry with respect to which those cells are really the same.

(vi) Investigate symmetries of the 4x4 magic squares. Find two 4x4 magic squares that cannot be obtained, one from the other, by application of these symmetries.

(vii) Find all possible triples from of the numbers 1 through 9 whose sum is 15 (the order of the triple doesn’t matter, so just put them in increasing order). Now for each of the numbers  $n$ , write down how many of these triples it is involved in. Similarly, label the squares of the tic-tac-toe board using a, b, c, d, e, f, g, h, i. By each letter write the number of horizontal, vertical, or diagonal lines crossing through the square for that letter. Use this data to show why Pick 15 and Tic Tac Toe are really the same game.

### The project requirements are:

1. Read and understand the above listed topics from the textbook and related online articles. Explain them to me.
2. Write a detailed report explaining the above with examples.
3. Illustrate these concepts by implementing examples in Matlab.
4. Give a presentation to your classmates on your project.

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<sup>1</sup>Adapted from Robert Ellis, IIT