Dear Math 400 Students,

It has been a pleasure to teach you all this semester. Real Analysis is an elegant subject that showcases the beauty of mathematical foundations. I hope I was able to give you a bit of a glimpse of the elegant arguments and beautiful concepts in it. I hope you remember and appreciate the many deep and powerful ideas (and not just their formulaic consequences in Calculus).

I would consider this a successful course, if you had fair opportunities to learn new ideas/concepts and you were challenged to grow intellectually. If you gained confidence in your ability to read, understand, and write mathematical arguments, as compared to the beginning of the semester. And, if you feel that you can read, understand, and apply any other topic/ technique in Real Analysis that you might need later on in your career. There is so much more to discover and learn. We essentially reached the beginning of the 20th century in terms of development of Analysis. In the 20th century, Analysis went beyond foundations for calculus and laid the mathematical foundations of probability, stochastics, and more. I hope you have a strong foundation to build upon and learn these advanced topics.

What next?

1) You can take courses at IIT that complement and supplement Math 400.
- Look into Math 402, Math 500, Math 501, Math 519.
- Math 556, the course on Metric Spaces, has not been offered in a while but if it is then go for it. Maybe I will get a chance to update and teach it some day. Modern Metric spaces are studied and used in Computer science and Combinatorics.

2) You can do some self study based on your interests.
- Read Section 6.7 from the textbook.
- Read "Surprises and Counterexamples in Real Function Theory" by Rajwade and Bhandari. There are many books on counterexamples in Analysis and they are always fun to try and read. Construction of interesting functions has always been an important part of building a mathematical theory like Analysis. This book is quite comprehensive. For simpler such books, look into "CounterExamples: From Elementary Calculus to the beginnings of Analysis" by Bourchtein and Bourchtein, or even simpler "Paradoxes and Sophisms in Calculus" by Klymchuk and Staples.
- Read Chapter 9 from the textbook.
- Look through Michael Spivak's "Calculus on Manifolds". Even though it's a very terse book and not what I would call a good textbook, it gets to the core of the main ideas very quickly. You can supplement it with a proper textbook like Munkres' "Analysis on Manifolds", or a more standard textbook like Apostol's "Mathematical Analysis".

3) Have Fun :-)
- Read Part III of each of the HWs from the course, if you haven't already done so. There are many interesting questions and recommendations there.
- Read "Proofs from the BOOK" by Aigner and Ziegler (several editions up 6th), which has many chapters on beautiful analytic results and arguments.
- Read "Godel, Escher, Bach: An Eternal Golden Braid" by Douglas Hofstadter. A beautiful book about foundations of mathematics and computer science.
- Read "MVT: Most Valuable Theorem" by Smorynski, an excellent historical and pedagogical study of the Mean Value Theorem.
- Read "Reverse Mathematics, proofs from the inside out" by Stillwell, if you want an in-depth study of how proofs depend on axioms.

- Look up my talk on Brouwer Fixed Point Theorem: http://www.math.iit.edu/~kaul/talks/IIT-MFT-Brouwer-Sperner.pdf
- Read articles from the Quanta Magazine. For example,
https://www.quantamagazine.org/tag/continuum-hypothesis
https://www.quantamagazine.org/tag/foundations-of-mathematics
https://www.quantamagazine.org/tag/applied-math

And there is so so much more to explore and discover. Don't hesitate to ask. I am always available if you need any help learning mathematics at any time.

I hope to see you in my classes, or at least hear from you, in the future.

best wishes,
Hemanshu