What is this course *really* about?
aka My aim for this course

"*It is impossible to define combinatorics, but an approximate description would go like this. We are given the job of arranging certain objects or items according to a specified pattern. Some of the questions that arise include:*
Is the arrangement possible?
In how many ways can the arrangement be made?
How do we go about finding such an arrangement?"
- Peter J. Cameron

"*We may loosely describe [combinatorics] as the branch of mathematics concerned with selecting, arranging, constructing, classifying, and counting or listing things. Much of combinatorics originated in recreational pastimes. But in recent years the subject has developed in depth and variety and has increasingly become a part of mainstream mathematics. Prestigious mathematical awards such as the Fields Medal and the Abel Prize have been given for groundbreaking contributions to the subject, while a number of spectacular combinatorial advances have been reported in the national and international media.*"
- Robin J. Wilson

"*Examples are the life blood of combinatorics. Careful study and exploration of examples has developed into many and varied areas of research in combinatorics – and we know that our examples can lead you to learn about, appreciate, and enjoy the combinatorial world.*"
- Ezra Brown and Richard K. Guy

"*If you ask a mathematician what they love most about mathematics, certain answers invariably arise: beauty, abstraction, creativity, logical structure, connection (between disciplines and between people), elegance, applicability, and fun. Combinatorics is the branch of mathematics best situated to embody and illustrate all of these virtues.*"
- Stephen Melczer

"*Combinatorics is everything. All our worlds, the physical, mathematical, and even spiritual, are inherently finite and discrete, and so-called infinities, be their actual or potential, as well as the 'continuum', are 'optical illusions'.*"
- Doron Zeilberger

According to Underwood Dudley, there are at least eight levels of mathematical understanding:

1. Being able to do arithmetic
2. Being able to substitute numbers in 'formulas'/ being able to state or use elementary properties of concepts
3. Given 'formulas'/ elementary properties of a concept, being able to get other 'formulas'/ elementary properties
4. Being able to understand hypotheses and conclusions of theorems
5. Being able to understand the proofs of theorems, step by step

6. Being able to *really* understand proofs of theorems: that is, seeing why the proof is as it is, and comprehending the underlying ideas of the proof and its relation to other proofs and theorems

7. Being able to generalize and extend theorems

8. Being able to see new relationships, and discover and prove entirely new theorems.

The word 'theorem' is used above in a very general sense - it can also represent algorithms and techniques with a mathematical basis.

Levels 5 and 6 would be considered basic mathematical ability for Math majors. Non-trivial applications of Mathematics would lie in-between levels 6 and 7. While levels 7 and 8 constitute research in Mathematics. A lot of engineering and physics is deep applied mathematics and requires understanding at or beyond levels 6 and 7.

Calculus courses focus on a mixture of 1 and 2. Math 230 (Introduction to Discrete Mathematics) focuses on 3 and 4. Math 332 (Elementary Linear Algebra) focuses on 3 and 4 with a bit of 5.

In this course (Math 453), the focus is more on the upper part of levels 3, 4, 5, and 6. The aim is give you a firm foundation in levels up to 6, so that you can go onto levels 7 and 8, as mathematical scientists who use and apply discrete mathematics. Combinatorial arguments and thinking pervades all parts of modern mathematics, computer science, physics, and biology, and this course gives you a gentle glimpse into that world.

I hope this course will help you make progress through these levels of mathematical understanding, and mathematical maturity. I would consider this a successful course, if you gain confidence in your ability to read, understand, and write proofs, especially as compared to the beginning of the semester. And, you feel that you can read, understand, and apply any other counting technique in combinatorics that you might need later on in your career.

with best wishes,
Hemanshu Kaul